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# CURVATURE MEASURES FOR PIECEWISE LINEAR MANIFOLDS[1]

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Let $K$ be a convex cell of dimension $m$ in Euclidean $n$-space, $R^n$. The volume of the tubular neighborhood of radius $\rho$ around $K$ is given by a polynomial, in $\rho$,

$$\sum_p \sum_i H^p(K_p^i) \frac{v_{n-m}}{v_{m-p}} \frac{\rho^{n-p}}{n-p} \int_{c_p^i} dS^{m-p-1},$$

where $H^p$ is the $p$-dimensional Hausdorff measure in $R^n$, $dS^k$ is the volume element of the standard unit sphere in $R^k$, $v_k$ is $H^{k-1}(S^{k-1})$, $K_p^i$ is a face of dimension $p$, $c_p^i$ is the outer normal angle determined by $K_p^i$, $p$ varies from 0 to $m$, $i$ varies from 1 to $N_p =$ the number of faces of dimension $p$, and $m < n$.

From this formula we can define the *pth curvature measure* of $K$ as follows. For any bounded Borel set $A \subset R^n$,

$$\sigma_p(A) = \sum_i H^p(A \cap K_p^i) \frac{1}{v_{m-p}} \int_{c_p^i} dS^{m-p-1}.$$

In addition to being measures, the $\sigma_p$ are invariant under the full Euclidean group of rigid motions in $R^n$ and satisfy the following strong stability property.

THEOREM 1. *Let $L$ be a $k$-dimensional affine subspace of $R^n$ and $\xi(n, k)$ the volume element of the manifold $E(n, k)$ of all $k$-dimensional affine subspaces in $R^n$. Then*

$$\int_{L \cap K \neq \varnothing} \sigma_j(L \cap K)\xi(n, k) = c_j \sigma_{n-k+j}(K),$$

*where $c_j$ is a constant depending on $n$, $m$, $k$.*

Given a piecewise linear manifold $K$ of dimension $m$, with boundary $\partial K$, piecewise linearly embedded in $R^n$ one can also define the $p$th curvature measure of $K$. For any bounded Borel set $A \subset R^n$,

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