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UNITARY WHITEHEAD GROUP OF CYCLIC GROUPS

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In topology, one encounters certain groups known as surgery obstruction groups, introduced by C. T. C. Wall [7]. They can be described in a purely algebraic setting, and so hopefully be computed by algebraic means, notably by techniques developed by H. Bass [1], which has come to be known as algebraic K-theory. This note aims at applying such techniques to the computation of the so-called unitary Whitehead groups, certain quotients of which are the Wall's surgery groups mentioned above. We only state the results and sketch the proof of the main theorem. Details will appear elsewhere. This work constitutes part of Chapter I of the author's dissertation [6] submitted to Columbia University. I am deeply indebted to my adviser, Professor Hyman Bass, for his extraordinary patience, generous help and inspiring guidance.

A unitary ring is a triple (A, λ, Λ), where A is a ring with involution denoted by $a \mapsto \bar{a}$, λ is an element in the center of A satisfying $\lambda\bar{\lambda} = 1$, and Λ is an additive subgroup of A satisfying the conditions

$$S_{-\lambda}(A) = \{a - \lambda\bar{a} | a \in A\} \subset \Lambda \subset \{a \in A | a + \lambda\bar{a} = 0\} = S^{-\lambda}(A)$$

and

$$\bar{a}ra \in \Lambda$$

whenever $a \in A, r \in \Lambda$. A morphism $f : (A, \lambda, \Lambda) \to (A', \lambda', \Lambda')$ between two unitary rings is a ring homomorphism $f : A \to A'$ satisfying the conditions $f(\lambda) = \lambda', f(\bar{a}) = \overline{f(a)}$ for all $a \in A$ and $f(\Lambda) \subset \Lambda'$. By an epimorphism (of unitary rings) we mean a morphism with $f(A) = A'$ and $f(\Lambda) = \Lambda'$. When A has trivial involution (that is, $a = \bar{a}$ for all $a \in A$, so that A has to be commutative), we single out the case $(A, 1, 0)$ and call it the orthogonal case. The symbol $U^{\lambda}_{2n}(A, \Lambda)$ will denote the group of all invertible $2n \times 2n$-matrices $\sigma = \left(\begin{smallmatrix}\alpha & \beta \\ \gamma & \delta\end{smallmatrix}\right)$ such that

$$\sigma^{-1} = \begin{pmatrix}\delta^* & \lambda\beta^* \\ \bar{\lambda}\gamma^* & \alpha^*\end{pmatrix}$$

where * means conjugate transpose, and $\beta\alpha^*$, $\delta\gamma^*$ have diagonal entries in Λ. The symbol $U^{\lambda}(A, \Lambda)$ will denote the group inj lim $U^{\lambda}_{2n}(A, \Lambda)$ with respect to the filtering given by

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