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REGULAR O (n)-MANIFOLDS, SUSPENSION OF KNOTS, AND KNOT PERIODICITY

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1. **Statement of the main results.** It will be convenient for us to define an n-knot to be a smooth, connected, oriented, n-dimensional (closed) submanifold Σ^n of S^{n+2} (oriented). If Σ^n is homeomorphic to S^n, then we call it a *spherical* knot. All manifolds in this note will be oriented and all constructions we consider will induce canonical orientations. This will be understood and not commented upon further.

Let K_n denote the semigroup of isotopy classes of smooth n-knots (S^{n+2}, Σ^n). Our object is to define a homomorphism

$$\omega: K_n \to K_{n+2}$$

which we think is reasonable to call "suspension". This homomorphism ω takes some spherical knots to nonspherical knots and vice-versa. (In fact, $\omega(S^{n+2}, \Sigma^n)$ is just a canonically defined embedding of the cyclic double covering of S^{n+2} branched at Σ^n in S^{n+4}.) However, if we iterate ω twice we obtain the following result:

THEOREM A. *The double suspension $\omega^2: K_n \to K_{n+4}$ takes homology spherical knots to spherical knots. Moreover, it induces a homomorphism $\omega^2: C_n \to C_{n+4}$ of (spherical) knot-cobordism groups, which is an isomorphism for $n \neq 1, 3$, an epimorphism for $n = 1$, and a monomorphism onto a subgroup of index two for $n = 3$. Also, ω^2 takes doubly null-cobordant knots to doubly null-cobordant knots.*

That such a homomorphism $C_n \to C_{n+4}$ exists was shown by Levine [7] because of his calculation of these groups, but our result gives the first explicit geometrically defined description of such a homomorphism. (Another, quite different, description has been concurrently and independently discovered by Cappell and Shaneson.)

The first statement in the theorem is an elementary consequence of the construction. The other statements follow from the following stronger facts: Let (S^{n+2}, Σ^n) be an n-knot and let $W^{n+1} \subset S^{n+2}$ be a Seifert surface spanning Σ^n. Then we construct canonically a Seifert surface $\omega(W^{n+1}) \subset S^{n+4}$ for the suspended knot $\omega(S^{n+2}, \Sigma^n)$. We prove that we can regard

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