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REAL CLOSURES OF SEMILOCAL RINGS, AND EXTENSION OF REAL PLACES

BY MANFRED KNEBUSCH

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(A) All rings in this announcement are commutative and with 1. For any ring K we denote by $W(K)$ the Witt ring of nondegenerate symmetric bilinear forms over K.

DEFINITION 1. A *signature* σ of K is a ring homomorphism from $W(K)$ to $\mathbf{Z}$.

REMARK 1. If K is a field, the signatures correspond uniquely with the orderings of K [**3**], [**9**]. Thus Theorem 1 below generalizes the main results of Artin-Schreier's theory of ordered fields [**1**].

We consider pairs (K, σ) with K a connected ring and σ a signature of K. There is an obvious notion of a homomorphism $(K, \sigma) \to (L, \tau)$ between pairs. We say that a homomorphism $\alpha: K \to L$ of rings is a (connected) *covering*, if α is the inductive limit of finite etale connected extensions of K, as studied in Galois theory. We say that a homomorphism $\alpha:(K, \sigma) \to (L, \tau)$ is a covering, if $K \to L$ is a covering.

DEFINITION 2. A *real closure* of a pair (K, σ) is a covering $\alpha:(K, \sigma) \to (R, \rho)$ such that (R, ρ) does not admit any coverings except isomorphisms.

By Zorn's lemma any pair (K, σ) has at least one real closure.

THEOREM 1. *Assume* $\alpha:(K, \sigma) \to (R, \rho)$ *is a real closure of a pair* (K, σ) *with* K *semilocal. Let* K_s *denote the universal covering* (= *separable closure*) *of* K.

(1) *For any other real closure* $\alpha':(K, \sigma) \to (R', \rho')$ *there exists an isomorphism* $\beta:(R, \rho) \xrightarrow{\sim} (R', \rho')$ *with* $\alpha' = \beta \circ \alpha$.

(2) *There does not exist any automorphism of* (R, ρ) *leaving all elements of* K *fixed except the identity.*

(3) *The Galois group of* K_s/R *is a 2-group.*

Assume in addition that 2 is a unit in A. Then even the following statements are true:

(3a) $K_s = R(\sqrt{-1})$.

(4) *If* R'/K *is any covering such that* $[K_s:R'] = 2$, *then* $K_s = R'(\sqrt{-1})$ *and* $W(R') \cong \mathbf{Z}$. *In particular* R' *has a unique signature.*

Thus if K *is semilocal with 2 a unit the signatures of* K *correspond uniquely with the conjugacy classes of involutions in the Galois group of* K.

REMARK 2. If K is a Dedekind domain at least statement (1) of Theorem 1 remains true and $[K_s:R] \leqq 2$.

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