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A HOMOTOPY CLASSIFICATION OF 2-COMPLEXES WITH FINITE CYCLIC FUNDAMENTAL GROUP

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For an arbitrary positive integer n, let Z_n denote the cyclic group of order n, and let $P_n = S^1 \cup_n e^2$ be the pseudo-projective plane of order n.

THEOREM. *Let X be a connected finite 2-dimensional CW-complex with fundamental group Z_n. Then*

(1) *X has the homotopy type of the sum $P_n \vee S^2 \vee \cdots \vee S^2$ of the pseudo-projective plane P_n and rank $H_2(X)$-copies of the 2-sphere S^2.*

(2) *There is a homotopy equivalence $f:X \to P_n \vee S^2 \vee \cdots \vee S^2$ realizing any prescribed Whitehead torsion $\tau(f) \in \mathrm{Wh}(Z_n)$.*

The result (1) was established in the prime order case by W. H. Cockcroft and R. G. Swan [**3**]. The work of P. Olum on the self-equivalences of the pseudo-projective plane P_n ([**6**], [**7**]) shows that every element of the Whitehead group Wh (Z_n) is realized as the torsion of some self-equivalence $P_n \to P_n$, so that (2) is a consequence of (1).

COROLLARY. *For connected finite 2-dimensional CW-complexes with finite cyclic fundamental group, homotopy type and simple homotopy type coincide.*

This generalizes to the nonprime order case a recent observation of W. H. Cockcroft and R. M. F. Moss [**2**].

SKETCH OF A PROOF OF THE THEOREM. Each CW-complex under consideration has the simple homotopy type of a complex P that is modeled in an obvious fashion on some presentation $\mathscr{P} = \langle a_1, \ldots, a_k : r_1, \ldots, r_m\rangle$ ($m \geqq k$) of the cyclic group Z_n. There are Nielsen transformations which reduce such a presentation to one of pre-Abelian form [**5**, p. 140]

$$\mathscr{Q} = \langle b_1, \ldots, b_k : b_1 W_1, \ldots, b_{k-1} W_{k-1}, b_k^n W_k, W_{k+1}, \ldots, W_m\rangle,$$

where the exponent sum of each word W_i with respect to each generator b_j is zero. Moreover, this Nielsen reduction $\mathscr{P} \to \mathscr{Q}$ corresponds to a simple homotopy equivalence $P \to Q$ of the associated topological models. Associated with each topological model P of a presentation $\mathscr{P}$ is the cellular chain complex $C_*(\tilde{P})$ of its universal covering $\tilde{P}$; the chain groups are free Z_n-modules which we give preferred bases according to a

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