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SUBMERSIONS FROM SPHERES

BY R. ESCOBALES, JR.

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1. Let M and B be Riemannian manifolds with M connected and complete. Assume π and $\bar{\pi}$ are Riemannian submersions from M onto B so that the fibers of these two submersions are totally geodesic. π and $\bar{\pi}$ are said to be *equivalent* provided there exists an isometry f of M which induces an isometry $\underline{f}$ of B so that the following diagram is commutative.

$$\begin{array}{ccc} M & \xrightarrow{f} & M \\ \pi \downarrow & & \downarrow \bar{\pi} \\ B & \xrightarrow{\underline{f}} & B \end{array}$$

We call the pair $(f, \underline{f})$ a *bundle isometry* of π and $\bar{\pi}$. Now set $\pi = \bar{\pi}$. π is *homogeneous* if for every $p, q \in M$ there exists a bundle isometry $(f, \underline{f})$ of π with $f(p) = q$.

In what follows S^m denotes the unit m sphere while $S^q(r)$ denotes a q sphere of radius r. $K_*(P_X Y)$ denotes the curvature of a 2 plane in B spanned by X and Y.

For a Riemannian submersion $\pi: M \to B$, O'Neill [**10**] has defined a tensor A which we call the *integrability tensor* of π. If $A \equiv 0$, then the horizontal distribution (the distribution complementary to the fibers in the tangent space of M) is integrable. In general we will follow the notation of [**10**]. We now state our first result. Complete proofs are found in [**4**].

THEOREM 1.1. *Let $\pi: S^m \to B$ be a Riemannian submersion with totally geodesic fibers. Assume $1 \leqq \dim$ fiber $\leqq m - 1$. Then as a fiber bundle π is one of the following types*:

$$\text{(a)}\quad \begin{array}{l} S^1 \to S^{2n+1} \\ \qquad\ \downarrow \pi \\ \qquad\ Cp(n) \quad \textit{for } n \geqq 2 \end{array} \qquad\qquad \text{(b)}\quad \begin{array}{l} S^3 \to S^{4n+3} \\ \qquad\ \downarrow \pi \\ \qquad\ Qp(n) \quad \textit{for } n \geqq 2 \end{array}$$

$$\text{(c)}\quad \begin{array}{l} S^1 \to S^3 \\ \qquad\ \downarrow \pi \\ \qquad\ S^2(\tfrac{1}{2}) \end{array} \qquad \text{(d)}\quad \begin{array}{l} S^3 \to S^7 \\ \qquad\ \downarrow \pi \\ \qquad\ S^4(\tfrac{1}{2}) \end{array} \qquad \text{(e)}\quad \begin{array}{l} S^7 \to S^{15} \\ \qquad\ \downarrow \pi \\ \qquad\ S^8(\tfrac{1}{2}) \end{array}$$

In cases (a) and (b) B is isometric to complex projective n-space and quaternionic n-space respectively with $1 \leqq K_*(P_X Y) \leqq 4$. In cases (c), (d) and (e) B is isometric to a sphere of curvature 4.

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