

COMPACT HILBERT CUBE MANIFOLDS AND THE INVARIANCE OF WHITEHEAD TORSION

BY T. A. CHAPMAN¹

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ABSTRACT. In this note we prove that every compact metric manifold modeled on the Hilbert cube Q is homeomorphic to $|K| \times Q$, for some finite simplicial complex K . We also announce an affirmative answer to the question concerning the topological invariance of Whitehead torsion for compact, connected CW-complexes. As a corollary of this latter result it follows that two compact Hilbert cube manifolds are homeomorphic iff their associated polyhedra (in the sense above) have the same simple homotopy type.

1. Introduction. In this note we announce some recent results on infinite-dimensional manifolds which imply, among other things, the topological invariance of Whitehead torsion for compact connected CW-complexes.

A *Hilbert cube manifold* (or Q -manifold) is a separable metric space which has an open cover by sets which are homeomorphic to open subsets of the Hilbert cube Q . We say that a Q -manifold X can be *triangulated* (or is *triangulable*) provided that X is homeomorphic (\cong) to $|K| \times Q$, for some countable locally-finite simplicial complex K . In [5] it was shown that (1) any open subset of Q is triangulable, and (2) if X is any Q -manifold, then $X \times [0, 1)$ is openly embeddable in Q and thus is triangulable (where $[0, 1)$ is the half-open interval). We refer the reader to [4] for a list of earlier results on Q -manifolds. In this note, based on results in [6], we prove that every compact Q -manifold can be triangulated. The triangulation of non-compact Q -manifolds is much more delicate and is expected to be the subject of a future paper.

TRIANGULATION THEOREM. *Every compact Q -manifold can be triangulated.*

Using a result of West [13] it follows that if K is any finite simplicial complex, then $|K| \times Q \cong M \times Q$, for some finite-dimensional combinatorial manifold M . In this sense it follows that all compact Q -manifolds can be *combinatorially triangulated*.

COROLLARY 1. *Every compact Q -manifold can be combinatorially triangulated.*

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