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COBORDISM OF $U(n)$-ACTIONS

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0. **Introduction.** Let G be a compact Lie group acting on a C^∞ manifold. A G invariant stable complex structure on M is a complex structure J on $T(M) \oplus \varepsilon^r$ (where ε^r is a trivial bundle) such that for each g in G, dg $\oplus$ id commutes with J. We will be concerned with the case where $G = U(n)$ and the action is free or regular. We will study the resulting bordism theories.

DEFINITION. Let M be a compact $U(n)$ manifold. M is called a regular $U(n)$ manifold if

1. Every isotropy group is conjugate to $U(k)$ for some $0 \leqq k \leqq n$.
2. For some r, $T(M) \oplus \varepsilon^r$ has a $U(n)$ invariant complex structure J such that the representation of the isotropy group $U(n)_x$ at $(T(M) \oplus \varepsilon^r)_x$ is equivalent to a sum of copies of the standard complex representation of $U(n)_x$ plus a trivial complex representation. (Remark. If $U(n)_x = g^{-1}U(k)g$, then $U(n)_x$ acts in the obvious way on $g^{-1}C^k \subset C^n$ and this is the standard representation.)

We define homotopy and equivalence classes of such structures analogously to [2]. The resulting bordism theory is denoted by $\Omega U(n)_*$. We denote the bordism theory of free $U(n)$-actions by $\Omega_*^{(n)}$. The main results are summarized in the following theorem.

THEOREM. *$\Omega_*^{(n)}$ and $\Omega U(n)_*$ are free MU_* modules. Any connected regular $U(n)$ manifold on which $U(n)$ acts nontrivially is bordant in $\Omega U(n)_*$ to a regular $U(n)$ manifold in which every isotropy group is conjugate to $U(1)$ or $U(0)$.*

Warning. $\Omega_*^{(n)}$ is not obviously $MU_*(BU(n))$.

1. **Relation between $\Omega_*^{(n)}$ and $\Omega U(n)_*$.** As in [3], [7] we construct a long exact sequence $\rightarrow D^{*,i} \rightarrow D^{*,i-1} \rightarrow E^{*,i-1} \rightarrow D^{*,i} \rightarrow \cdots$ and a resulting exact couple and a spectral sequence. Then E^∞ is associated to a filtration of $\Omega U(n)_*$. For $k \neq n$, $E^1_{*,k}$ is the bordism group of pairs (E, M) where E is a complex $U(n)$ vector bundle over the regular $U(n)$ manifold M such that every point in M has isotropy group conjugate to $U(n-k)$ and the representation of $U(n)_x$ on E_x is a sum of copies of the standard complex representation of $U(n)_x$. The pair (E, M) is completely determined by the $U(n-k) \times U(k)$ manifold M_0, the points in M with isotropy group

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