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A NOTE ON POINCARÉ 2-COMPLEXES

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The purpose of this note is to announce some progress on the following conjecture:

CONJECTURE. Every Poincaré 2-complex is of the homotopy type of a closed 2-manifold.

By connected Poincaré n-complex we mean a connected CW complex X dominated by a finite CW complex which satisfies Poincaré duality with local coefficients: Let $\pi = \pi_1 X$ and let $\Lambda = Z\pi$ be the group ring of π. Let $\omega : \pi \to \{\pm 1\}$ be a homomorphism (trivial if X is to be "oriented"). Let $\bar{\Lambda}$ be the right π-module whose elements are the same as Λ but the right action is given as follows: For $\lambda \in \bar{\Lambda}$, $x \in \pi$, $\lambda \cdot x = \omega(x)x^{-1}\lambda$. Then there exists some class $[X] \in H_n(X; Z \otimes_\Lambda \bar{\Lambda})$ such that $[X] \cap : H^i(X; \Lambda) \to H_{n-i}(X; \bar{\Lambda})$ is an isomorphism for all i.

Wall's results [1] give the following ($\simeq$ means "homotopy equivalent to"):

THEOREM (WALL). *Let X be a connected Poincaré 2-complex. Let $\pi = \pi_1 X$. Then*

(a) *if π is finite, $X \simeq S^2$ or RP^2;*

(b) *if π is infinite then X is a $K(\pi, 1)$;*

(c) *there exists a unique 2-manifold M_X such that $H_*(X; Z) \simeq H_*(M_X; Z)$ (simple coefficients);*

(d) *$X \simeq X'$ a CW complex of dimension $\leqq 3$.*

Thus the conjecture becomes, more specifically: If X is a Poincaré 2-complex, then $X \simeq M_X$. The results we have obtained so far are the following:

THEOREM. *Let X be a connected finite Poincaré 2-complex; then*

(a) *if $M_X = S^2$ or RP^2 then $X \simeq M_X$,*

(b) *if X is 2-dimensional as a CW complex and $M_X = S^1 \times S^1$ or the Klein bottle, then $X \simeq M_X$.*

In both (a) and (b) the unoriented case follows from the oriented: If $M_X = RP^2$ (resp. the Klein bottle), then it can be shown that X', a certain double cover of X, is a Poincaré 2-complex with $M_{X'} = S^2$ (resp. $S^1 \times S^1$). Assuming the oriented case, we get $X' \simeq S^2$ (resp. $S^1 \times S^1$).

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