BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 78, Number 5, September 1972

UNIFORM ESTIMATES FOR THE $\bar{\partial}$-EQUATION ON INTERSECTIONS OF STRICTLY PSEUDOCONVEX DOMAINS[1]

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Communicated by Lee Rubel, April 10, 1972

1. A few years ago Grauert-Lieb [5] and Henkin [3] solved independently the $\bar{\partial}$-problem with uniform bounds for (0, 1)-forms on strictly pseudoconvex domains in $\boldsymbol{C}^n$ with smooth boundary, i.e., they proved that for every bounded $\bar{\partial}$-closed $C^\infty_{(0,1)}$-form f on such a domain D there is a bounded C^∞ function u on D with $\bar{\partial}u = f$. In the proofs, the solution u is constructed explicitly in terms of integrals involving a holomorphic kernel constructed by Henkin [2] and Ramirez [10]. Kerzman [7] extended this idea to obtain a local version of the above result, which enables him to get the same result for strictly pseudoconvex domains with a smooth boundary in a Stein manifold. Based on results of Koppelman [8], Lieb [9] then obtained uniform estimates for the $\bar{\partial}$-equation for $(0, q)$-forms on the same class of domains. Recently Henkin [4] announced the solution of the $\bar{\partial}$-problem with uniform bounds for (0, 1)-forms on certain analytic polyhedra.

In this note we announce the solution of the $\bar{\partial}$-problem with uniform bounds for $(0, q)$-forms on a domain which is the intersection of a finite number of strictly pseudoconvex domains intersecting normally. As a consequence of this result, for any complex manifold, one can find a locally finite open covering such that the $\bar{\partial}$-equation can be solved with uniform bounds on intersections of members of the covering. This essentially answers a question raised by Gunning [6, p. 73].

2. To state our result precisely, let D be a bounded domain in $\boldsymbol{C}^n$ such that there exist a finite open covering $\{U_j\}_{j=1}^k$ of ∂D and strictly plurisubharmonic C^4 functions $\rho_j: U_j \to \boldsymbol{R}$, $1 \leqq j \leqq k$, such that $D \cap (\bigcup_{j=1}^k U_j)$ is the set of all $x \in \bigcup_{j=1}^k U_j$ which, for every $1 \leqq j \leqq k$, satisfy $x \notin U_j$ or $\rho_j(x) < 0$. Set $S_j = \{x \in U_j \cap \partial D: \rho_j(x) = 0\}$. We assume that for every sequence $1 \leqq i_1 < \ldots < i_l \leqq k$, $1 \leqq l \leqq 2n$, and for every $x \in \bigcap_{v=1}^l S_{i_v}$ the 1-forms $d\rho_{i_1}, \ldots, d\rho_{i_l}$ are linearly independent over $\boldsymbol{R}$ at x.

AMS 1969 *subject classifications*. Primary 3215, 3222; Secondary 3250.
Key words and phrases. Integral representations, $\bar{\partial}$ estimates, pseudoconvex domains, cohomology with bounds.

[1] Research partially supported by National Science Foundation Grant GP-27624.

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