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FILTERED AND ASSOCIATED GRADED RINGS

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1. **Introduction.** The object of this note is to present a condition which guarantees that a filtered ring A is isomorphic (in the category of filtered rings) to its associated graded ring gr A. The result is that a separated, complete, nonnegatively filtered ring A over a field k of characteristic 0 is isomorphic to gr A if and only if $\dim_k H^2(\text{gr } A, \text{gr } A) = \dim_k H^2(A, A)$ where the $\dim_k H^2(\text{gr } A, \text{gr } A)$ is finite. The tool is algebraic deformation theory. Rim has observed that an application of the main theorem yields a condition for a plane algebroid curve over an algebraically closed field of characteristic 0 to be of the form $u^m = v^n$—a result obtained by Zariski [**5**] by a different approach.

2. Since A is a deformation of gr A (Gerstenhaber [**1**]), there exists a one-parameter family of deformations $A_t = \text{gr } A[[t]]$ with multiplication defined by $f_t(a, b) = ab + tF_1(a, b) + t^2F_2(a, b) + \cdots$. It is known that the deformation from gr A to A given by A_t is a "pop deformation", i.e., for $t \neq 0$, A_t is isomorphic as a filtered ring to $A[[t]]$ (Gerstenhaber [**2**]).

Let δ_t denote the Hochschild coboundary operator of the algebra A_t, i.e., computed relative to the multiplication f_t. For example, for $\varphi \in C^1(A_t, A_t)$, the group of 1-cochains of A_t, one has

$$\delta_t\varphi(a, b) = f_t(a, \varphi b) - \varphi(f_t(a, b)) + f_t(\varphi a, b).$$

If there exists $\eta_t \in C^1(A_t, A_t)$ such that $z_t = \delta_t\eta_t$, then $z_t \in B^2(A_t, A_t)$. $z_0 \in Z^2(\text{gr } A, \text{gr } A)$ is *extendible* if there exists $z_t \in Z^2(A_t, A_t)$ such that

$$z_t = z_0 + tz_1 + t^2z_2 + t^3z_3 + \cdots.$$

Note that every $b_0 \in B^2(\text{gr } A, \text{gr } A)$ is extendible since $b_0 = \delta\eta_0$ implies that $b_t = \delta_t\eta_0 = b_0 + tb_1 + t^2b_2 + \cdots$ is an extension of b_0 where η_0 is extended linearly over $k((t))$. An *extendible class* of $H^2(\text{gr } A, \text{gr } A)$ is a $[z_0]$ for which there is a representative z_0 which is extendible. $z_0 \in Z^2(\text{gr } A, \text{gr } A)$ is a *jump cocycle* if there exists an extension z_t of z_0 such that $z_t \in B^2(A_t, A_t)$. Each $b_0 = \delta\eta_0 \in B^2(\text{gr } A, \text{gr } A)$ is a jump cocycle since $b_t = \delta_t\eta_0$ is an extension of b_0 and $b_t \in B^2(A_t, A_t)$. A *jump class* of $H^2(\text{gr } A, \text{gr } A)$ is a $[z_0]$ for which there exists a representative z_0 which is a jump cocycle.

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[1] The results announced here are contained in the author's Ph.D. thesis. written under the guidance of Murray Gerstenhaber at the University of Pennsylvania.

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