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PROPAGATION OF ANALYTICITY FOR SOLUTIONS OF DIFFERENTIAL EQUATIONS OF PRINCIPAL TYPE

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Let $P = P(x, D)$ be a linear differential operator of order m with analytic coefficients in the open set $X \subset R^n$, such that

$$d_\xi P_m(x, \xi) \neq 0, \quad \text{when } (x, \xi) \in Z = \{(x, \xi) \in X \times R^n \backslash \{0\}; P_m(x, \xi) = 0\}.$$

We shall also assume that P satisfies one of the following three conditions:

(a) The principal part $P_m(x, D)$ of P is real.

(b) $d_\xi \operatorname{Re} P_m$ and $d_\xi \operatorname{Im} P_m$ are linearly independent in Z and the Poisson bracket $\{\operatorname{Re} P_m, \operatorname{Im} P_m\}$ vanishes there.

(c) $P = P(D)$ has constant coefficients.

THEOREM 1. *Suppose that P satisfies either* (a), (b) *or* (c). *Then the analytic wave front set* (*see* [**4**]) *of a distribution u, such that Pu is analytic, is a union of entire bicharacteristic strips.*

REMARK 1. In the case (b) the bicharacteristic strips are two-dimensional submanifolds of Z generated by the Hamilton fields $H_{\operatorname{Re} P_m}$ and $H_{\operatorname{Im} P_m}$ and for operators satisfying (c) they are linear manifolds of the form $\{(x + d_\xi \operatorname{Re}(\alpha P_m(\xi)), \xi); \alpha \in C\}$ for some x and some ξ with $P_m(\xi) = 0$. Here their dimension may vary between 1 and 2.

REMARK 2. For the local version, in $X \times R^n\backslash\{0\}$, of the theorem we need of course only to make the assumptions on P locally.

REMARK 3. When P satisfies (a) *and* (c) the corresponding regularity theorem in X was proved in [**1**]. For arbitrary operators satisfying (a) the more precise result concerning analytic wave front sets has been proved by Hörmander [**4**] and, in the framework of hyperfunctions, by Kawai-Kashiwara (see [**5**]). For the case (b) Kawai has announced (private correspondence) that by extending the theory of Fourier integral operators to the analytic category, he and Kashiwara have proved the result of this note. (Probably under the weaker assumption that $H_{\operatorname{Re} P_m}$, $H_{\operatorname{Im} P_m}$ and the cone axis are linearly independent in Z.)

The following definition is a slight variation of the definition given in [**4**] of the analytic wave front set $WF_a(u)$ of a distribution u.

DEFINITION 1. $(x^0, \xi^0) \notin WF_a(u)$ if and only if there is a neighborhood U

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