BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 78, Number 3, May 1972

COTERMINAL FAMILIES AND THE STRONG MARKOV PROPERTY

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Communicated by Harry Kesten, October 27, 1971

1. Let Ω be the set of right continuous paths mapping $[0, \infty)$ into (E_Δ, ρ), where E is a locally compact, separable metric space with metric ρ and $E_\Delta = E \cup \{\Delta\}$, $\{\Delta\}$ the point at infinity. Following the notation of [**1**], we let $X = (\Omega, X_t, \mathscr{F}, \mathscr{F}_t, \theta_t, P^x)$ be a strong Markov process with stationary transition probabilities on (E_Δ, ρ), and we use $\mathscr{B}$ and $\mathscr{B}_\Delta$ to denote the σ-fields of Borel sets on E and E_Δ respectively. The σ-fields $\{\mathscr{F}_t, 0 \leqq t\}$ are assumed to be right continuous. Finally we assume a fixed initial distribution μ, and all a.s. statements will refer to $P = \int_{E_\Delta} \mu(dx)P^x$.

The purpose of this note is to outline a type of strong Markov property for a class of random times, resembling last exit times from sets. To motivate the problem, write the strong Markov property at a stopping time T as

$$P(X_t \in A \mid \mathscr{F}_T) = P^{X_T}(X_{t-T} \in A) \quad \text{a.s.} \tag{1}$$

on $\{T < t\}$, where $\mathscr{F}_T$ is the usual σ-field of information up through T. We are interested in finding an analogue of (1) with T replaced by random times such as L^t, the last exit before $(t + 0)$ from a given set, and $\mathscr{F}_T$ replaced by an appropriate σ-field. Since the conditioning now involves the future of the process, the distributions on the right of (1) will be altered but will be seen to depend only on X_{L^t} or X_{L^t-} and $t - L^t$.

As an example suppose X is reflecting Brownian motion on $[0, \infty)$, $P = P^0$ and L^t is the last hit of $\{0\}$ prior to t. We leave it to the reader to verify that if $\sigma(L^t)$ is the σ-field generated by L^t, then

$$P^0(X_t \in A \mid \sigma(L^t)) = \int_A \frac{y \exp(-y^2/2(t - L^t))\, dy}{t - L^t} \quad \text{a.s.} \tag{2}$$

on $\{0 < L^t < t\}$. Moreover another computation shows that

$$\begin{aligned} &\int_0^\infty \frac{1}{\sqrt{s}}\left(\frac{y \exp(-y^2/2s)\, dy}{s}\right) P^y(T_{\{0\}} > t, X_t \in A) \\ &\qquad = \int_A \frac{1}{\sqrt{s+t}}\left(\frac{x \exp(-x^2/2(t+s))\, dx}{s+t}\right), \end{aligned} \tag{3}$$

AMS 1970 *subject classifications*. Primary 60J25, 60J40.