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THE $\mathcal{S}$-MATRIX ASSOCIATED WITH NONSELFADJOINT DIFFERENTIAL OPERATORS[1]

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1. Let $q(x)$ denote a complex-valued potential defined in R^N, N-dimensional Euclidean space ($N \geqq 1$). Suppose that $q(x)$ satisfies the following condition:

(C) $q(x) \in L_2^{\text{loc}}(R^N)$ and there exist constants $\alpha > 0$, $\rho > 0$, such that

$$\max_{|x|>\rho} |q(x)|e^{\alpha|x|} < \infty.$$

Let A_0 (A) denote the selfadjoint (closed) operator acting in $H = L_2(R^N)$ given by $-\Delta$ ($-\Delta + q(x)\cdot$). It is our intention in this note to study the $\mathcal{S}$-matrix, $\mathcal{S}(\kappa)$ ($\mathcal{S}'(\kappa)$), associated with the operators A_0 and A (A^*).

In two previous papers, [1] and [2], we derived an abstract scattering theory for two operators A_0 and A acting in a Hilbert space H, where A_0 is selfadjoint and A is closed. We also showed in [2] that these results are applicable to the operators A_0 and A defined above.[2] To be more precise, we considered the operators $A_{0_\mathcal{G}} = A_0E_{0_\mathcal{G}}$ and $A_\mathcal{G} = AE_\mathcal{G}$, where $\{E_{0_\lambda}\}$ denotes the spectral resolution for the selfadjoint operator A_0, $E_\mathcal{G}$ is a projection operator and $\mathcal{G}$ is a closed subinterval of $(0, \infty)$, satisfying the following condition:

($C_\mathcal{G}$) There exists no nontrivial outgoing or incoming solution of the equation $(-\Delta - \lambda + q(x))u(x) = 0$ for any λ in $\mathcal{G}$.[3]

In [1] we established the existence of "wave operators," $W^\pm$ ($W'^\pm$) and the scattering operator, $S = W^{+-1}W^-$ ($S' = W'^{+-1}W'^-$), associated with $A_{0_\mathcal{G}}$ and $A_\mathcal{G}$ ($A^*_\mathcal{G}$) using a stationary formulation. From this, we obtained the similarity of $A_{0_\mathcal{G}}$ and $A_\mathcal{G}$ ($A^*_\mathcal{G}$). In [2] we expressed $W^\pm$ ($W'^\pm$) in terms of a time-dependent formulation. S was expressed in terms of "distorted plane waves" by means of the "$\mathcal{S}$-matrix" (see §2).

In this paper, we shall obtain a meromorphic continuation of the $\mathcal{S}$-matrix and distorted plane waves from the interval $\mathcal{G}$ to a strip in the

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[2] In [2], the condition on $q(x)$ was weaker than (C). However, we shall need the exponential decay in order to obtain the results of §2.

[3] By outgoing (incoming), we mean that $u(x)$ satisfies the outgoing (incoming) radiation condition: $u(x) = O(|x|^{(1-N)/2})$ and $(\partial/\partial|x| - i\lambda^{1/2})u(x) = o(|x|^{(1-N)/2})$ $((\partial/\partial|x| + i\lambda^{1/2})u(x) = o(|x|^{(1-N)/2}))$ as $|x| \to \infty$.