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ABSTRACT WAVE EQUATIONS WITH FINITE VELOCITY OF PROPAGATION[1]

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Let G be a locally compact abelian group. Let $L^2(G)$ be the Hilbert space of all complex-valued functions on G that are measurable and square-integrable with respect to Haar measure on G. Let B be a self-adjoint translation-invariant operator in $L^2(G)$, not necessarily bounded, and consider the abstract wave equation

$$d^2u/dt^2 + B^2u = 0$$

where u is a function from the nonnegative real axis to $L^2(G)$. For any ϕ and ψ in $L^2(G)$, a solution is given by

$$u(t) = (\cos tB)\phi + \left(\frac{\sin tB}{B}\right)\psi, \tag{*}$$

in the sense that

$$\frac{d^2}{dt^2}\langle u(t), w\rangle + \langle u, B^2w\rangle = 0$$

for all w in the domain of B^2, where $\langle\cdot, \cdot\rangle$ denotes the usual inner product in $L^2(G)$. In the classical case in which G is an n-dimensional Euclidean space $\mathbf{R}^n$ and $-B^2$ is the Laplacian, the solution (*) has the property that if ϕ and ψ have compact support then so does $u(t)$ for all $t>0$. In fact, there exists a compact subset K_t of $\mathbf{R}^n$, independent of ϕ and ψ, such that supp $u(t)\subset(\text{supp }\phi\cup\text{supp }\psi)+K_t$, where supp f denotes the support of f. Our first theorem says that on $\mathbf{R}^n$ this is essentially the only operator B for which the abstract wave equation has this property, which we call finite velocity of propagation. Recall now that if B is a selfadjoint translation-invariant operator in $L^2(G)$, then there must exist a real measurable function β on the dual group Γ of G such that $(Bf)\hat{} = \beta\hat{f}$, where $\hat{f}$ denotes the Fourier transform of f.

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