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ON THE SCHUR SUBGROUP

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Let A_x be a simple component of the group algebra QG of a finite group G over the rationals Q having center K. Let p be a rational prime and Q_p the p-adic completion of Q. Then $Q_p \otimes_Q A_x$ is a direct sum of simple algebras $K_{\mathfrak{p}} \otimes_K A_x$ where $\mathfrak{p}$ is a prime divisor of p. The characters associated with these simple components are all conjugate. Since if a representation with K-valued character χ can be written in $K_{\mathfrak{p}}$, then a representation with character χ^σ may be written in $K_{\mathfrak{p}\sigma}$, we see that either A_x is split at every prime dividing p or at none. This fact has also been observed independently by Mark Benard.

We are now in a position to apply results of Mac Lane [3, Corollary] together with those of [1]:

THEOREM 1. *If A_x is a quaternion algebra central over K, then $A_x \sim K \otimes_L B$ where L is any subfield of K such that the galois group of K/L is cyclic, and B is a simple algebra central over L.*

Hence if K/Q is cyclic of odd order, the simple algebras central over K appearing in some QG are precisely those of the form $K \otimes_Q B$ where B is a quaternion algebra over Q (cf. [2]).

As a final remark, we observe that the above together with the construction of [1] enable us to determine those algebras of index 3 in Sch $(Q\sqrt{(-3)})$, the Schur subgroup of $Q(\sqrt{(-3)})$. By [2], they are *not* of the form $Q(\sqrt{(-3)})\otimes_Q A$. Hence by Mac Lane [3], the above remarks and the following construction, they must have zero Hasse invariant at any primes which are ramified or inertial; i.e. if $p \equiv -1 \bmod (3)$ or $p=3$ then there is zero Hasse invariant at primes of $Q(\sqrt{(-3)})$ extending these. If $p\equiv 1$ (3), then p splits into 2 primes in $Q(\sqrt{(-3)})$, and we show that there is an A_x with Hasse invariant $\frac{1}{3}$ at one of these primes, $\frac{2}{3}$ at the other, and zero elsewhere: Simply let G be the group generated by a, b, c where $a^p=1$, $b^{p-1}=c$, $c^3=1$ and where c is central and b acts on a as the generator of the galois group

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