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BOUNDARY BEHAVIOR OF HARMONIC FUNCTIONS ON HERMITIAN HYPERBOLIC SPACE[1]

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Let $D=\{z=(z_1, \cdots, z_n)\in C^n : h(z)=\operatorname{Im} z_1-\sum_2^n |z_k|^2>0\}$, and $B=\partial D=\{z : h(z)=0\}$. Writing $z_j=x_j+iy_j$ we let β be the measure on B given by $d\beta=dx_1dx_2dy_2 \cdots dx_ndy_n$. D is a Siegel domain of Type II which is the image of the unit ball $D=\{z\in C^n:\sum_1^n |z_k|^2<1\}$ under the generalized Cayley transform:

$$z_1 \mapsto i\frac{1+z_1}{1-z_1}, \quad z_k \to \frac{iz_k}{1-z_1}, \qquad k=2, \cdots, n.$$

Let N be the group of holomorphic automorphisms of D consisting of the elements $(a, c)\in R\times C^{n-1}$ acting on D in the following way:

$$(a, c): z_1 \to z_1 + a + 2i\sum_{k=2}^n z_k\bar{c}_k + i\sum_{k=2}^n |c_k|^2,$$

$$(a, c): z_k \to z_k + c_k, \qquad k \geqq 2.$$

N acts simply transitively on B. We will consider real-valued functions on D which are harmonic with respect to the Laplace-Beltrami operator:

$$L = h(z)\left\{4y_1\frac{\partial^2}{\partial z_1\partial\bar{z}_1} + \sum_2^n \frac{\partial^2}{\partial z_k\partial\bar{z}_k} + 2i\sum_2^n \bar{z}_k\frac{\partial^2}{\partial z_1\partial\bar{z}_k} - 2i\sum_2^n z_k\frac{\partial^2}{\partial\bar{z}_1\partial z_k}\right\}.$$

In [2] Korányi defined the following notion of admissible convergence in D: let us call

$$\Gamma_\alpha(u) = \left\{z\in D:\operatorname{Max}\left[\,|\operatorname{Re} z_1 - \operatorname{Re} u_1|,\ \sum_2^n |z_k-u_k|^2\right] < \alpha h(z),\ h(z)<1\right\}$$

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