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THE MANY-ONE EQUIVALENCE OF SOME GENERAL COMBINATORIAL DECISION PROBLEMS

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1. **Introduction.** A *decision problem* for a combinatorial system shall denote a pair (ϕ, S) where ϕ is a specified kind of decision problem (e.g. derivability problem, halting problem, etc.) and S is a combinatorial system. Two decision problems (ϕ_1, S_1), (ϕ_2, S_2) are said to be of the same *many-one degree* (of unsolvability) if there exist effective many-one mappings f and g such that each instance of (ϕ_1, S_1) is reducible to an instance of (ϕ_2, S_2) via f and each instance of (ϕ_2, S_2) is reducible to an instance of (ϕ_1, S_1) via g.

A *general combinatorial decision problem*, i.e., a decision problem for a class of combinatorial systems, shall denote a pair (ϕ, C) where ϕ is a specified kind of decision problem and C is a class of combinatorial systems (e.g. Turing machines, semi-Thue systems, etc.). A general combinatorial decision problem (ϕ_1, C_1) is *many-one reducible* to another general combinatorial problem (ϕ_2, C_2) if there exists an effective one-one mapping ψ of the problems p associated with (ϕ_1, C_1) into the problems associated with (ϕ_2, C_2) such that p is of the same many-one degree as $\psi(p)$. (ϕ_1, C_1) and (ϕ_2, C_2) are said to be *many-one equivalent* if each is many-one reducible to the other.

The reduction of one general combinatorial decision problem to another has been investigated by numerous authors. In particular, W. E. Singletary [15] has combined results of his own and those of others in such a way as to provide an effective proof of the (r.e.) equivalence of a number of general combinatorial decision problems. This former work has lead W. W. Boone to suggest that a stronger form of equivalence might exist between at least some subset of the problems considered. Our aim is to show that a number of these general problems are many-one equivalent. In addition, we indicate that these are, in a sense, best possible results.

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