BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 77, Number 3, May 1971

THE CLASSIFICATION OF FREE ACTIONS OF CYCLIC GROUPS OF ODD ORDER ON HOMOTOPY SPHERES

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Communicated by M. F. Atiyah, June 19, 1970

Consider the cyclic group Z_p acting freely on a homotopy sphere Σ^{2n+1}. This action is a map $\mu: Z_p \times \Sigma \to \Sigma$. We shall consider the three cases where the action is smooth, piecewise linear (PL) or topological. If we pick a generator $T \in Z_p$ then we may say that two actions $\mu_i: Z_p \times \Sigma_i^{2n+1} \to \Sigma_i^{2n+1}$, $i=1, 2$, are equivalent actions if there is an equivalence $f: \Sigma_1 \to \Sigma_2$ in the appropriate category (i.e. f is diffeomorphism, PL equivalence or homeomorphism depending on whether μ_i are smooth, PL or topological) such that $f\mu_1(T, x) = \mu_2(T, f(x))$. Among smooth actions we have the linear actions on $S^{2n+1} \subset C^{n+1}$ where the action is given by

$$(z_0, \cdots, z_n) \to (\exp[2\pi i\theta_0/p]z_0, \cdots, \exp[2\pi i\theta_n/p]z_n)$$

where $\theta_0, \cdots, \theta_n$ are integers between 1 and $p-1$, prime to p. The quotient of S^{2n+1} by this action is the Lens space $L^{2n+1}(p; \theta_0, \cdots, \theta_n)$, and it is well known that for any free Z_p action on Σ^{2n+1}, the orbit space Σ/Z_p is homotopy equivalent to $L^{2n+1}(p; \theta_0, \cdots, \theta_n)$ for some appropriate choice of $\theta_0, \cdots, \theta_n$.

Now two Z_p-actions (μ_i, Σ_i), $i=1, 2$, are equivalent if and only if the orbit spaces Σ_i/Z_p are equivalent in the appropriate category. The equivalence is given by an orientation preserving isomorphism $g: \Sigma_1/Z_p \to \Sigma_2/Z_p$ such that $g_*(T) = T$ where T is identified with the corresponding element in $\pi_1(\Sigma_i/Z_p)$. Thus the problem of equivalence of Z_p actions is the problem of classifying manifolds of the homotopy type of Lens spaces up to equivalence in the corresponding category.

In this note we announce a procedure for doing this when p is odd, $n \geqq 2$. Some related work has been done by R. Lee [3], [4]. The result for $p=2$ was previously found by Lopez de Medrano [5] and Wall [11] and the answer for p odd is quite analogous. One uses the theory of surgery to study the problem, and calculates as far as possible the terms arising. As usual one arrives at a complete classification in the

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AMS 1970 *subject classifications*. Primary 57D65, 57E05; Secondary 20C10.

Key words and phrases. Free cyclic group actions on spheres, surgery, free action, Lens space, simple homotopy structure, signature invariant, torsion, suspension.

[1] Partially supported by an NSF grant.