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ON THE ANALYSIS AND SYNTHESIS OF CERTAIN ABSTRACT SYSTEMS

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ABSTRACT. The systems considered here are those describable as continuous functions mapping one compact metric space into another. A class of such systems is determinable if the members can be distinguished to within a given error by a finite number of elementary measurements made on the domain and range spaces. Such classes are necessarily relatively compact in the uniform metric and admit a common modulus of continuity. This announcement provides quantitative estimates of the number of measurements required in terms of the common modulus of continuity of the class, and the metric entropy, capacity, and dimension of the domain and range spaces. These data are used to construct uniform simplicial and polynomial approximations for each member of the class, and to provide quantitative estimates for the degrees of the approximating polynomials required.

For many purposes a physical system can be usefully modeled as a transfer function of prescribed continuity taking one metric space (the input space) into another (the output space). Within this framework, an analysis of the system is an approximate description of this function in terms of data taken from the input and output spaces, while a synthesis of the system is an approximate realization of the function in terms of known elementary functions. One primary purpose of systems studies is to provide effective analysis and synthesis procedures for rendering heretofore intractable systems accessible to modern computing machinery. We announce here some recent quantitative results bearing on the complexity of such procedures; details will appear elsewhere [1].

We denote by X and Y the *input* and *output* spaces, respectively. We suppose that both X and Y are furnished with a metric, which provides a measure of the distinguishability of elements in each of these spaces. We assume that the elements of both X and Y can be approximated to within any prescribed error by a finite number of elements; this assumption forces both X and Y to be (relatively) *compact* metric spaces [2].

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