

## CODIMENSION-ONE FOLIATIONS OF SPHERES

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Let  $M$  be an  $n$ -dimensional, differentiable manifold with a (possibly empty) boundary  $\partial M$ . A *smooth, codimension-one foliation* of  $M$  is a decomposition of  $M$  into disjoint, connected subsets, called the *leaves* of the foliation, with the following properties:

(i) At each point  $p \in M$  there exist local  $C^\infty$ -coordinates  $(x_1, \dots, x_n)$  such that in a neighborhood of  $p$  the leaves are described by the equations  $x_n = \text{constant}$ .

(ii) Each component of  $\partial M$  is a leaf.

In 1951 George S. Reeb constructed a smooth, codimension-one foliation of  $S^3$  [4], and it has since been shown by Lickorish [2], and independently by Novikov and Zieschang, that, in fact, every compact, orientable 3-manifold can be so foliated. By using the polynomial  $p(Z_0, Z_1, Z_2) = Z_0^3 + Z_1^3 + Z_2^3$  in complex 3-space and the theorems in [3], we prove the following:

**THEOREM 1.** *There exists a smooth, codimension-one foliation of  $S^5$  having one compact leaf  $B$  such that:*

(a)  *$B$  is diffeomorphic to  $S^1 \times L$  where  $L$  is a circle bundle over a 2-torus,  $T^2$ .*

(b) *All the noncompact leaves of one component of the foliation are diffeomorphic to  $R^2 \times T^2$ .*

(c) *All the noncompact leaves of the other component have the homotopy-type of a bouquet  $S^2 \vee \dots \vee S^2$  of eight 2-spheres.*

By using Theorem 1 and an inductive procedure, we then establish

**THEOREM 2.** *There exist smooth, codimension-one foliations of each of the spheres  $S^{2^k+3}$  for  $k=1, 2, 3, \dots$ . (The sequence begins:  $S^5, S^7, S^{11}, S^{19}, S^{35}, \dots$ )*

**COROLLARY 1.** *For  $n=2^k+1$ ,  $k=1, 2, 3, \dots$ , there exist smooth, codimension-one foliations of the manifolds  $D^2 \times S^n$  and  $D^2 \times V_{n+1,2}$  where  $V_{n+1,2} = \text{SO}(n+1)/\text{SO}(n-1)$ .*

**COROLLARY 2.** *For  $n=2^k+4$ ,  $k=1, 2, 3, \dots$ , there exist smooth,*

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