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A REMARK ON CLASSIFICATION OF RIEMANNIAN MANIFOLDS WITH RESPECT TO $\Delta u = Pu$

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Consider an orientable C^∞ Riemannian manifold R of dimension $m \geqq 2$ and an elliptic partial differential equation $\Delta u = Pu$ on R with P a nonnegative, not identically zero, C^1 function on R. We denote by O_{PX} the set of pairs (R, P) such that the subspace $PX(R)$ of the space $P(R)$ of solutions u of $\Delta u = Pu$ on R determined by a property X reduces to $\{0\}$. Here the possibilities for X that we consider are B (boundedness), D (Dirichlet-finite: $D_R(u) = \int_R du \wedge * du < \infty$), E (energy-finite: $E_R(u) = \int_R (du \wedge * du + Pu^2 * 1) < \infty$), and their combinations BD, BE. The purpose of this note is to announce the following complete inclusion relations:

$$O_G < O_{PB} < O_{PD} = O_{PBD} < O_{PE} = O_{PBE}. \tag{1}$$

The symbol $\mathfrak{A} < \mathfrak{B}$ means that $\mathfrak{A}$ is a proper subset of $\mathfrak{B}$ and O_G is the set of pairs (R, P) such that R does not possess a harmonic Green's function. This type of classification of Riemannian manifolds was initiated by Ozawa [9]. Myrberg [3] demonstrated the existence of the Green's function of $\Delta u = Pu$ on R and thus also the existence of a positive solution of $\Delta u = Pu$ on R without any further restrictions on P. One of the most interesting results of Ozawa is:

$$O_{PB} = O_{PD} = O_{PBD} = O_{PE} = O_{PBE} \tag{2}$$

if the pairs (R, P) are required to satisfy $\int_R P * 1 < \infty$. This condition was weakened by Glasner-Katz [1] as follows: (2) is valid for (R, P) such that there exists a subregion $R' \notin SO_{HD}$ (cf. [11]) with $\int_{R'} P * 1 < \infty$. That this is the weakest condition under which (2) is valid was shown by Glasner-Katz-Nakai [2].

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