BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 77, Number 3, May 1971

ORIENTATION-PRESERVING MAPPINGS, A SEMIGROUP OF GEOMETRIC TRANSFORMATIONS AND A CLASS OF INTEGRAL OPERATORS[1]

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Communicated by H. S. M. Coxeter, September 29, 1970

Let A, B be smooth ($=C^\infty$), oriented n-manifolds, A with naturally oriented boundary, ∂A, and B without boundary.

A very important problem in geometric analysis is that of giving an algebraic and/or combinatorial characterization of those smooth mappings from ∂A to B which can be extended to a smooth, orientation-preserving mapping from A to B.

In this work, one such characterization is given in the particular case where A is the unit disk, D ($\partial D = S^1$), and B is the plane, R^2. An application is made to a class of convolution-type operators to show they are topologically equivalent to the Hilbert transform.

1. **Preliminaries.** A smooth $f: S^1 \to R^2$ is called *extendable* if there is a smooth $F: D^- \to R^2$ (D^- closure of D) with nonnegative Jacobian, J_F, and whose restriction to S^1 is f. If, further, $J_F > 0$ on S^1 then f is *properly extendable.*

A *Titus transformation* T is a linear operator on the vector space of smooth functions from S^1 to R^2 given by:

$$(Tf)(t) = f(t) + c(t) \det[v, f'(t)]v, \tag{1.1}$$

c a nonnegative, smooth function on S^1. The set of all finite compositions of Titus transformations is a semigroup, $\mathfrak{T}$. The effect of a Titus transformation can be represented by an elementary operation of growth along a fixed direction, growth understood in the sense of moving to the outside of an oriented curve.

A "degenerate" mapping $f: S^1 \to R^2$ is one whose image lies in a one-dimensional subspace. A *Titus mapping* (T-mapping) is the image by an element of $\mathfrak{T}$ of a degenerate mapping. A Titus mapping, thus, has

AMS 1970 *subject classifications.* Primary 57D40, 47D05, 44A35; Secondary 30A90, 47E05, 44A15.

Key words and phrases. Normal immersions, extendable mappings, holomorphic mappings, Hilbert transform.

[1] This research is contained in the author's doctoral dissertation submitted to the University of Michigan, and was supported by a Scholarship from Conselho Nacional de Pesquisas (Brazil). The author wishes to thank his advisor, Professor Charles Titus, for suggesting the problem and for his assistance throughout the work.