BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 77, Number 3, May 1971

# ON INSEPARABLE GALOIS THEORY

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Communicated by Alex Rosenberg, November 23, 1970

Throughout this note $k$ will be a field of characteristic $p \neq 0$, and $K$ will be a modular extension of $k$ [9]; i.e., a finite purely inseparable field extension of $k$ which is a tensor product, over $k$, of primitive extensions. We shall outline a Galois theory of modular field extensions which, for the special case where the exponent of $K/k$ is one, reduces to the well-known Galois correspondence of Jacobson [5, p. 186] between intermediate fields in the extension and restricted Lie subalgebras of $L(K/k) = \mathrm{Der}_k(K, K)$ which are also $K$-subspaces ($L(K/k)$ being the restricted Lie $k$-algebra and $K$-space of derivations of $K$ over $k$).

There have recently appeared in the literature a number of other approaches to inseparable Galois theory, in varying stages of development; see, e.g. Sweedler [8], [9], Shatz [7], Davis [2], Gerstenhaber and Zaromp [3]. Our treatment utilizes the Hopf algebraic techniques of [8].

**1. Basic concepts.** A cocommutative $k$-coalgebra $C$ [10, p. 63] will be called a *divided power coalgebra* if $[C:k]$ is a power of $p$ and $C \approx C_1 \otimes_k \cdots \otimes_k C_r$, where each coalgebra $C_i$ is spanned by a sequence of divided powers [10, p. 268]. A *divided power Hopf algebra* is a Hopf $k$-algebra which is a divided power coalgebra. The $k$-space $P(C)$ of primitive elements of $C$ [10, p. 199] is a restricted Lie $k$-algebra [4] if $C$ is a Hopf algebra, the Lie multiplication and $p$-power map in $P(C)$ being defined by the formulae $[x, y] = xy - yx$ and $x^{[p]} = x^p$ for $x$, $y$ in $C$.

THEOREM 1. *There exists a divided power Hopf $k$-algebra $H(K/k)$ and a measuring $\omega_{K/k}: H(K/k) \otimes K \to K$ [10, p. 138] with the following universal property. Given any measuring $\omega: C \otimes K \to K$, with $C$ a divided power $k$-coalgebra, there is a unique coalgebra map $f: C \to H(K/k)$ such that $\omega = \omega_{K/k}(f \otimes 1_K)$. $H(K/k)$ is uniquely determined by $K/k$ up to Hopf algebra isomorphism, and $[H(K/k):k] = [K:k]^{[K:k]}$. Moreover, there exists a restricted Lie algebra isomorphism $P(H(K/k)) \approx L(K/k)$,*

*AMS* 1970 *subject classifications*. Primary 12F15; Secondary 16A24.

*Key words and phrases*. Inseparable field extensions, approximate automorphisms, higher derivations, Hopf algebras.

[1] Supported in part by NSF GP-9395. The results of §1 were presented to the Society on May 14, 1969.