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A RENEWAL THEOREM FOR DISTRIBUTIONS ON R^1 WITHOUT EXPECTATION[1]

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ABSTRACT. Let $U\{I\}$ be the expected number of visits to an interval I of a random walk associated with a distribution on R^1 in the domain of attraction of a stable law with exponent $\frac{1}{2}<\alpha\leqq 1$. Theorem A gives asymptotic expressions for $U\{I\pm t\}$ as $t\to\infty$. Such expressions are not valid when $0<\alpha\leqq\frac{1}{2}$ without additional hypotheses on F. These are indicated in Theorem B.

1. Theorem 1 of [3] extends to distributions on all of R^1 as follows: (Notation as in [3] or [4, Chapter XI].) Let F be a probability distribution on $(-\infty, \infty)$ and for any measurable set I put

$$U\{I\} = \sum_{n=0}^{\infty} F^{n*}\{I\}$$

finite or not. As in [3] we assume F is nonarithmetic. (See note (iv) in §2 below.)

THEOREM A. *Suppose*

$$1 - F(t) + F(-t) = t^{-\alpha}L(t), \qquad t > 0, \tag{1}$$

and

$$\lim_{t\to\infty} \frac{F(-t)}{1-F(t)} = \frac{q}{p} \tag{2}$$

where $0<\alpha\leqq 1$, $p+q=1$ *and* L *is slowly varying at* ∞. *Then when* $\frac{1}{2}<\alpha<1$,

$$\lim_{t\to\infty} t^{1-\alpha}L(t)(U\{I+t\} + U\{I-t\}) = \frac{\sin \pi\alpha}{\pi(p^2 + 2pq\cos\pi\alpha + q^2)} |I| \tag{3}$$

and

$$\lim_{t\to\infty} \frac{U\{I-t\}}{U\{I+t\}} = \frac{q}{p} \tag{4}$$

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