NOT EVERY 0-DIMENSIONAL REALCOMPACT
SPACE IS N-COMPACT

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Introduction. N-compact spaces were introduced by S. Mrówka in [1], where the general concept of an E-compact space was defined: given a Hausdorff space E, a space X is E-*compact* if it is homeomorphic to a closed subspace of E^m for some cardinal number m. Thus the I-compact spaces (where I is the closed unit interval) are the compact Hausdorff spaces, the R-compact spaces are the realcompact spaces or Q-spaces, and the 2-compact spaces (where 2 represents the discrete space consisting of two points) are the 0-dimensional compact Hausdorff spaces. The N-compact spaces are those which can be embedded as closed subspaces in N^m where N is the set of natural numbers with the discrete topology.

In terms of category theory, the category of E-compact spaces forms the epireflective hull of E in T_2, the category of Hausdorff spaces, i.e., the intersection of all replete reflective subcategories of T_2 containing the space E (cf. [4]).

The main properties of E-compact spaces were given in [2], where it was asserted that the N-compact spaces are precisely the 0-dimensional realcompact spaces ("0-dimensional" there, as here, means "having a base of clopen sets"). It is clear that every N-compact space is 0-dimensional and realcompact, but the proof of the converse in [2] was incomplete and indeed the validity of the converse remained an open question.

The first significant result in that direction was obtained by H. Herrlich in [3], where he proved that every strongly 0-dimensional realcompact space is N-compact (X is strongly 0-dimensional iff it is completely regular and βX is totally disconnected). Attention then focused on the only known example of a 0-dimensional realcompact space which is not strongly 0-dimensional, a metrizable space Δ introduced by P. Roy in [5] and described in detail in [6]. The main result of this paper is that Δ is not N-compact; and so it is established that the class of N-compact spaces is properly contained in the class of