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FUNCTION ALGEBRAS AND THE DE RHAM THEOREM IN PL

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0. Introduction. There is a classical contravariant functor on the category of smooth manifolds M which assigns to each M the algebra A of all smooth functions on M, and one uses this functor implicitly throughout differential topology. For example, the de Rham theorem extends the customary derivation $d:A\to\mathcal{E}(A)$ to a cochain complex $(\Lambda\mathcal{E}(A), d)$ whose homology is isomorphic to the real cohomology of M itself. In this paper we construct a corresponding contravariant functor on the category of piecewise linear manifolds M, which assigns to each M an algebra A of functions on M. We then define a derivation $d:A\to\mathcal{E}(A)$ and extend it to a cochain complex $(\Lambda\mathcal{E}(A), d)$ whose homology is isomorphic to the real cohomology of M; this is the de Rham theorem in PL. As an application we construct connections and curvature homomorphisms in terms of $(\Lambda\mathcal{E}(A), d)$, to which we apply a real version of the Chern-Weil theorem to compute real Pontrjagin classes of PL manifolds without using the Hirzebruch L-polynomials.

1. Smoothing homeomorphisms. A *simplicial decomposition of* $\boldsymbol{R}^n$ *at* 0 is any finite triangulation of $\boldsymbol{R}^n$ into open simplexes such that $0\in\boldsymbol{R}^n$ is the only 0-simplex. If α and β are any two such simplicial decompositions then we write $\alpha<\beta$ whenever β is a subdivision of α. For any α and β there is a simplicial decomposition γ with $\alpha<\gamma$ and $\beta<\gamma$, so that the simplicial decompositions of $\boldsymbol{R}^n$ at 0 form a directed set.

It is clear that a simplicial decomposition α is completely determined by its 1-simplexes $\rho_1, \cdots, \rho_N$ (for some $N>n$), each p-simplex of α containing precisely p 1-simplexes $\rho_{i_1}, \cdots, \rho_{i_p}$ in its closure. If $\boldsymbol{R}^n$ is endowed with its usual euclidean norm then points on each 1-simplex ρ_i can be identified with their norms $x_i\in\boldsymbol{R}^+$, and points in the open p-simplex determined by $\rho_{i_1}, \cdots, \rho_{i_p}$ can be identified by the coordinates $(x_{i_1}, \cdots, x_{i_p})\in(\boldsymbol{R}^+)^p$.

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