BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 77, Number 3, May 1971

GENERATION OF EQUICONTINUOUS SEMIGROUPS BY HERMITIAN AND SECTORIAL OPERATORS. II

BY ROBERT T. MOORE[1]

Communicated by M. H. Protter, September 25, 1970

1. **Introduction.** This announcement concerns the topological aspects of the generation theory of equicontinuous semigroups and groups of operators on a complete complex locally convex space (lcs) $\mathfrak{X}$, and uses the recalibration theorem from [6] to relate these to the more geometrical aspects treated in [8] (with which the reader is assumed to be familiar). Perturbation techniques from [7], along with other devices, are used to develop applications to the theory of abstract heat equations and to the theory of distribution semigroups. Details will appear in [9].

2. **Quasi-equicontinuous semigroups.** The semigroups considered here generalize the contraction holomorphic semigroups $\mathrm{CH}(\Phi, \Gamma)$ on a complete complex lcs $\mathfrak{X}$ discussed in [8], where for, $0 \leqq \Phi \leqq \pi/2$, $S_\Phi = \{z \in \boldsymbol{C} : |\arg z| \leqq \Phi\}$ and $\Delta_\Phi = \{z \in \boldsymbol{C} : \pi/2 + \Phi \leqq \arg z \leqq 3\pi/2 - \Phi\}$.

DEFINITION 1. Let $\omega \geqq 0$. Then a family $\{T_z : z \in S_\Phi\} \subset \mathfrak{L}(\mathfrak{X})$ of continuous linear transformations is a *quasi-equicontinuous holomorphic semigroup of type* ω, or is in $\mathrm{EH}(\Phi; \omega)$ iff

(a) it satisfies the usual algebraic, continuity and holomorphy conditions as in Definition (1a) of [8], and

(b) the family $\{e^{-\omega z} T_z : z \in S_\Phi\}$ is equicontinuous in $\mathfrak{L}(\mathfrak{X})$.

EXAMPLES. (1) If $\{T_t : t \in [0, \infty)\}$ is a classical C_0 semigroup on a B-space [3], and $\omega > \omega_0 = \lim\{(t^{-1} \log \|T_t\|) : t \to \infty\}$, then $\|T_t\| \leqq Me^{\omega t}$ for suitable M and the semigroup is in $\mathrm{EH}(0; \omega)$ since operator-norm-bounded sets are equicontinuous. Similarly, every semigroup in Hille's class $H(-\Psi, \Psi)$ on a B-space [3] is in $\mathrm{EH}(\Phi; \omega(\Phi))$ for every $\Phi < \Psi$ and suitable $\omega(\Phi)$.

(2) Every $\mathrm{CH}(\Phi, \Gamma)$ semigroup from [8] is in $\mathrm{EH}(\Phi; 0)$.

(3) Every equicontinuous C_0 semigroup as in Yosida [10] is in

---

AMS 1969 *subject classifications.* Primary 4750, 4748; Secondary 4601.

Key words and phrases. C_0 equicontinuous semigroups, quasi-equicontinuous holomorphic semigroup, exponential distribution semigroup, contraction semigroup, infinitesimal generator, sectorial operator, hermitian operator, analytic continuation of semigroups, abstract Laplacian, abstract heat equation.

[1] The author gratefully acknowledges partial support of this research by the National Science Foundation through NSF GP 12548.