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FOURIER TRANSFORMS OF UNBOUNDED MEASURES[1]

BY LOREN ARGABRIGHT AND JESÚS GIL DE LAMADRID

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Let G be a locally compact abelian group. For a (generally unbounded) measure μ on G we shall say that μ is *transformable* if there is a measure $\hat{\mu}$ on the character group Γ of G such that, for every $f\in K(G)$, the space of continuous functions with compact support on $G, \hat{f}\in L_2(\hat{\mu})$ and

$$\int_G f^* * f(x)\, d\mu(x) = \int_\Gamma |\hat{f}(\gamma^{-1})|^2\, d\hat{\mu}(\gamma). \tag{1}$$

The resulting "Fourier transformation" $\mu\rightarrow\hat{\mu}$ contains the classical theory and leads to generalizations of a variety of classical results, including the Plancherel theorem and the Poisson summation formula. The present work can also be regarded as a sort of theory of tempered distributions on general locally compact abelian groups. It is true that Bruhat [**11**] introduced a direct generalization to this setting of the theory of Schwartz [**10**], but, to the best of our knowledge, a detailed study of the Fourier transform has not been carried out. In a forthcoming exposition we shall describe the precise relation between the present study and the work of Bruhat.

In the present announcement of results, we shall, for the most part, restrict ourselves to the study of the Fourier transform on the linear span $\mathfrak{M}(G)$ of all positive definite measures on G. Such a measure μ is defined by the property: $\mu(f * f^*) \geqq 0$, for $f\in K(G)$. The transformability of the elements of $\mathfrak{M}(G)$ follows from a general Plancherel theorem of Godement [**6**, p. 144–07].[2] If μ is the Dirac measure at the identity, Godement's theorem reduces to the classical Plancherel theorem for G. In a similar spirit, a general Poisson summation formula can be established for positive definite μ, which reduces to the classical formula in the case that μ is Haar measure on a closed subgroup of G. This is the content of Theorem 6 below.

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[2] We are indebted to the referee for bringing this reference to our attention. This made it possible to shorten a previous version of our announcement.