BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 77, Number 3, May 1971

A FIXED POINT THEOREM FOR PLANE CONTINUA[1]

BY CHARLES L. HAGOPIAN[2]

Communicated by Steve Armentrout, October 9, 1970

ABSTRACT. In this paper it is proved that every bounded arcwise connected plane continuum which does not separate the plane has the fixed point property.

A set X is said to have the fixed point property if each continuous function f on X into itself leaves some point fixed (that is, there is a point x belonging to X such that $f(x)=x$). The problem "Must a bounded plane continuum which does not separate the plane have the fixed point property?" has motivated a great deal of research in plane topology. K. Borsuk in 1932 proved that every Peano continuum which lies in the plane and does not separate the plane has the fixed point property [2]. Since that time, other general conditions have been found which insure that a plane continuum has this property. In 1967, H. Bell proved that every bounded plane continuum which does not separate the plane and has a hereditarily decomposable boundary has the fixed point property [1]. The following question is still outstanding. If a bounded plane continuum is arcwise connected and does not separate the plane, then must it have the fixed point property? Here an affirmative answer is given to this question by proving the following theorem. If M is a bounded arcwise connected plane continuum which does not have infinitely many complementary domains, then the boundary of M does not contain an indecomposable continuum.

Throughout this paper S is the set of points of a simple closed surface (that is, a 2-sphere).

The proof of the following theorem is based on techniques which are closely related to the folded complementary domain concept defined by F. Burton Jones [3, p. 173].

THEOREM 1. *Suppose M is a continuum in S, $S-M$ does not have*

AMS 1970 *subject classifications.* Primary 54H25, 57A05, 47H10, 55C20; Secondary 54F22.

Key words and phrases. The fixed point property, arcwise connected continua, folded complementary domain, plane continua which do not separate the plane.

[1] This research was partially sponsored by the Air Force Office of Scientific Research and the Sacramento State College Foundation.

[2] The author gratefully acknowledges conversations about topics in this paper with Professors C. H. Papas, E. Kymala, R. Leezer, and F. B. Jones.