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THE ENDOMORPHISMS OF CERTAIN ONE-RELATOR GROUPS AND THE GENERALIZED HOPFIAN PROBLEM

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Introduction. A great deal of progress has been made in the past decade in the theory of groups with a single defining relation which possesses elements of finite order [1], [2], [3]. G. Baumslag [2] has pointed out a class $\mathcal{L}$ of torsion-free nonhopfian one-relator groups, found in [4], that support the view that torsion is a simplifying rather than a complicating factor in the theory of one-relator groups. Here we characterize the endomorphisms of the groups in $\mathcal{L}$, compute the centralizers of certain special elements and use these results to prove:

If G is in $\mathcal{L}$ then there is a proper fully invariant subgroup N of G such that G/N is isomorphic to G.

Preliminaries. $\mathcal{L}$ consists of the groups

$$G(l, m) = (a, b; a^{-1}b^l a = b^m)$$

where $|l| \neq 1 \neq |m|$, $lm \neq 0$ and l, m are relatively prime. Let G' denote the normal closure of b in $G(l, m)$ and G'' the commutator subgroup of G'. For $n \neq 0$, let $A(n, p, q)$ denote the group

$$(X_p, \cdots, X_0, \cdots, X_q; X_p^l = X_{p+1}^m, \cdots, X_{q-1}^l = X_q^m)$$

where $-p$ and q are maximal nonnegative integers such that $l^q | n$, $m^{-p} | n$. We then have

LEMMA 1. *The map $F: F(a) = a$, $F(b) = b$ defines an onto endomorphism of $G(l, m)$ with nontrivial kernel N where N is the normal closure of the subgroup generated by*

$$W(a, b) = ([b, a]^t b^s) b^{-1} \quad \textit{and} \quad V(a, b) = a^{-1}ba([b, a]^t b^s)^{-m}$$

such that $(m - l)t + ls = 1$.

PROOF. F is onto but not 1-1 as found in [4]. The rest is a straightforward computation.

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