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A LECTURE ON THE GEOMETRY OF NUMBERS OF CONVEX BODIES

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Before I can explain the subject of my talk, let me introduce the notation to be used: R^n denotes the n-dimensional space of all points, $\mathbf{x}=(x_1, \cdots, x_n)$, $\mathbf{y}=(y_1, \cdots, y_n)$, $\mathbf{0}=(0, \cdots, 0)$, etc., with real coordinates, $\mathbf{0}$ being called the *origin*. Such points will be treated as vectors, and we put

$$\mathbf{x}+\mathbf{y} = (x_1+y_1, \cdots, x_n+y_n), \qquad C\mathbf{x} = (Cx_1, \cdots, Cx_n)$$

where C is any real number. We also use the inner product

$$\mathbf{x}\mathbf{y} = x_1y_1 + \cdots + x_ny_n$$

of two points and the determinant

$$(\mathbf{x}^{(1)}, \cdots, \mathbf{x}^{(n)}) = \begin{vmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & & \vdots \\ x_{n1} & \cdots & x_{nn} \end{vmatrix}$$

of n points

$$\mathbf{x}^{(h)} = (x_{h1}, \cdots, x_{hn}) \qquad (h = 1, 2, \cdots, n).$$

This determinant is $\neq 0$ if and only if the n points are linearly independent over R. Points with integral coordinates are called *lattice points*, and we use Λ to denote the lattice of all such lattice points. Λ is an Abelian group with n independent generators under addition. Every bounded set contains at most finitely many lattice points.

We shall be concerned with the relation between Λ and convex bodies. Here a convex body K is to mean a bounded closed convex set in R^n which contains the origin as an interior point and is symmetric in $\mathbf{0}$. Important examples are the "cube" $|x_1| \leqq 1, \cdots, |x_n| \leqq 1$, the "octahedron" $|x_1| + \cdots + |x_n| \leqq 1$, and the "sphere" $x_1^2 + \cdots + x_n^2 \leqq 1$. The volume of a convex body K is defined by

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