BOUNDARY VALUES OF HOLOMORPHIC FUNCTIONS

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Let $\mathfrak{D}$ be a bounded domain in C^n with smooth boundary. We shall consider the behavior near the boundary of holomorphic functions in $\mathfrak{D}$. Our results are of two kinds: those valid without any further restriction on $\mathfrak{D}$, and those which require that $\mathfrak{D}$ is strictly pseudo-convex. Detailed proofs will appear in [7].

1. **Fatou's theorem and H^p spaces.** We assume that $\mathfrak{D}$ is a bounded domain with smooth boundary. We first define the appropriate approach to the boundary which extends the usual nontangential approach and takes into account the complex structure of C^n. Let $w \in \partial\mathfrak{D}$, and let ν_w be the unit outward normal at w. For each $\alpha > 0$ consider the approach region $\mathfrak{A}_\alpha(w)$ defined by

$$\mathfrak{A}_\alpha(w) = \{z \in \mathfrak{D}: |(z - w, \nu_w)| < (1 + \alpha)\delta_w(z), \ |z - w|^2 < \alpha\delta_w(z)\}.$$

Here $(z, w) = z_1\bar{w}_1 + \cdots + z_n\bar{w}_n$, $|z|^2 = (z, z)$, and $\delta_w(z)$ denotes the minimum of the distances of z from $\partial\mathfrak{D}$ and from z to the tangent hyperplane to $\partial\mathfrak{D}$ at w.

We shall say that F is *admissibly bounded at* w if $\sup_{z\in\mathfrak{A}_\alpha(w)} |F(z)| < \infty$, for some α; F has an *admissible limit at* w, if $\lim_{z\to w,\, z\in\mathfrak{A}_\alpha(w)} F(z)$ exists, for all $\alpha > 0$. On $\partial\mathfrak{D}$ we shall take the measure induced by Lebesgue measure on C^n; we denote it by $m(\cdot)$, or $d\sigma$. The extension of the classical Fatou theorem is as follows.

THEOREM 1. *Suppose F is holomorphic and bounded in $\mathfrak{D}$. Then F has an admissible limit at almost every $w \in \partial\mathfrak{D}$.*

Note. This is stronger than the usual nontangential approach one would obtain using the theory of harmonic functions in R^{2n}. As is to be observed, the admissible approach allows a parabolic tangential approach in directions corresponding to $2n-2$ real dimensions.

We consider two types of balls on $\partial\mathfrak{D}$. For any $\rho > 0$ and $w \in \partial\mathfrak{D}$,

(1) $B_1(w, \rho) = \{w' \in \partial\mathfrak{D}: |w - w'| < \rho\}$;

(2) $B_2(w, \rho) = \{w' \in \partial\mathfrak{D}: |(w - w', \nu_w)| < \rho, \ |w - w'|^2 < \rho\}$.

Observe that $m(B_1(w, \rho)) \sim c_1\rho^{2n-1}$, and $m(B_2(w, \rho)) \sim c_2\rho^n$ as $\rho \to 0$.

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