SURFACES OF LEAST AREA WITH PARTIALLY FREE BOUNDARY ON A MANIFOLD SATISFYING THE CHORD-ARC CONDITION[1]

BY JOHANNES C. C. NITSCHE

Communicated by M. H. Protter, May 21, 1970

Consider a configuration in Euclidean 3-space consisting of a surface T and of a rectifiable Jordan arc $\Gamma = \{\mathfrak{x} = \mathfrak{z}(\tau);\ 0 \leqq \tau \leqq 1\}$ having its end points on T, but no other point in common with T. Denote by P the semidisc in the (u, v)-plane $P = \{u, v;\ u^2+v^2 < 1,\ v > 0\}$, by $\partial' P$ and $\partial'' P$ its boundary portions $\{u, v;\ u^2+v^2=1,\ v>0\}$ and $\{u, v;\ -1<u<1,\ v=0\}$, respectively, and by P' the domain $P \cup \partial' P$.

A surface $S = \{\mathfrak{x} = \mathfrak{x}(u, v);\ (u, v) \in P'\}$ is said to be bounded by the above configuration, or chain $\langle \Gamma, T\rangle$, if its position vector $\mathfrak{x}(u, v) = \{x(u, v), y(u, v), z(u, v)\}$ satisfies the following conditions:

(i) $\mathfrak{x}(u, v) \in C^0(P')$.

(ii) $\mathfrak{x}(u, v)$ maps the arc $\partial' P$ onto the open arc $(\Gamma) = \{\mathfrak{x} = \mathfrak{z}(\tau);\ 0<\tau<1\}$ monotonically in such a way that

$$\lim_{\theta \to +0} \mathfrak{x}(\cos\theta, \sin\theta) = \mathfrak{z}(0), \qquad \lim_{\theta \to \pi - 0} \mathfrak{x}(\cos\theta, \sin\theta) = \mathfrak{z}(1).$$

(iii) The relation $\lim_{n\to\infty} d_T[\mathfrak{x}(u_n, v_n)] = 0$ holds for every sequence of points (u_n, v_n) in P' converging to a point on $\overline{\partial'' P}$.

Here $d_T[\mathfrak{x}] = \inf_{\mathfrak{t} \in T} |\mathfrak{x} - \mathfrak{t}|$ denotes the distance between the point $\mathfrak{x}$ and the surface T.

Obviously, the convergence specified under (iii) is uniform in the following sense:

$$\lim_{\delta \to 0} \sup_{(u,v) \in P';\, 0 < v \leqq \delta} d_T[\mathfrak{x}(u, v)] = 0.$$

Thus while the distance function $d_T[\mathfrak{x}(u, v)]$ is continuous in $\overline{P}$, the same cannot generally be said about the vector $\mathfrak{x}(u, v)$. In fact, the trace of S on T, i.e. the set of limit points on T for all sequences $\mathfrak{x}(u_n, v_n)$ as in (iii) above, may well look quite bizarre. Examples illustrating such contingencies can be found in [2, pp. 95–96] and [4, pp. 220–222].

AMS 1970 *subject classifications.* Primary 49F10, 53A10; Secondary 26A63, 35B45, 49F25.

Key words and phrases. Surfaces of least area, minimal surfaces, surfaces minimizing Dirichlet's integral, free boundary value problem, least area property, free boundary, trace, regularity of free boundary, chord-arc condition, length-area principle, lift-off condition.

[1] This work was supported by Grant number AFOSR 883-67.