VECTOR FIELDS AND GAUSS-BONNET

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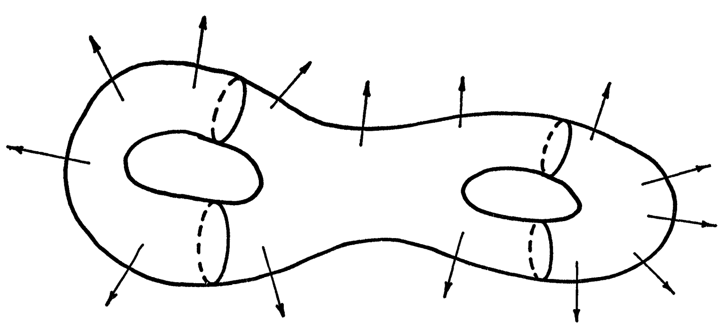
ABSTRACT. The topic is vector-fields and characteristic classes. The starting point is the classical Gauss-Bonnet theorem and the H. Hopf index theorem. After recalling these, curvature is used to define the Chern class of a complex analytic manifold. Then a recently proved formula relating Chern classes to zeroes of meromorphic vector-fields is given.

This expository note will briefly outline some recent developments involving zeroes of vector fields and characteristic classes. The characteristic classes used will be defined.

This really begins with the classical Gauss-Bonnet theorem [**17**], so recall this theorem. Let M be a smooth compact oriented surface (without boundary) in $\mathbf{R}^3$. $M \subset \mathbf{R}^3$. Let ν be a smooth field of unit normal vectors on M.



Assume that ν is compatible with the orientation of M in the sense that given $p \in M$ and given a positively oriented basis e_1, e_2 for

$$T_pM \ (T_pM = \text{tangent space of } M \text{ at } p),$$

then $\nu(p)$ is a positive multiple of $e_1 \times e_2$. Let S^2 be the unit sphere of $\mathbf{R}^3$. $S^2 = \{(x_1, x_2, x_3) \in \mathbf{R}^3 \mid x_1^2 + x_2^2 + x_3^2 = 1\}$. Take S^2 with its standard

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