THE EXACT SEQUENCE OF LOW DEGREE AND NORMAL ALGEBRAS

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Communicated by I. N. Herstein, February 18, 1970

The exact sequence of low degree associated to a first quadrant bicomplex (five terms long in [4, I.4.5.1] seven terms long in [2, Lemma 7.5]) has been used in a number of situations, for example, in obtaining a cohomological description of the Brauer group of a commutative ring R [2]. In this note we observe that the sequence may be extended to an infinitely long exact sequence. The terms arising from the homology of the total complex are not $F^{n-1}H^n(\text{tot})$, the $(n-1)$th filtration group of H^n, for $n>2$, but map onto it.

As an application we embed the seven term Galois cohomology sequence of [1, 5.5] into an infinite sequence, and sketch a map from normal Azumaya algebras into the eighth term which extends the Teichmüller cocycle map of [3].

1. Suppose given a bicomplex $\{C_{p,q}\}$ of abelian groups [5, p. 340] such that $C_{p,q}=0$ if $p<0$ or $q<0$. The differentials $d': C_{p,q}\to C_{p+1,q}$ and $d'': C_{p,q}\to C_{p,q+1}$ of the bicomplex, defined for all integers p, q, satisfy the conditions $d'd'=0$, $d''d''=0$, $d'd''+d''d'=0$. (Notation: cl( ) will denote "cohomology class of.") Then $Z^2_{p,q}$ is the set of classes $\text{cl}(u)$ in $\ker(d'')/\text{im}(d'')$ such that u is in $C_{p,q}$, $d''(u)=0$, and $d'(u)=d''(v)$ for some v in $C_{p+1,q-1}$; $B^2_{p,q}$ is the set of classes $\text{cl}(u)$ such that u in $C_{p,q}$ is of the form $u=d'(v)+d''(w)$ with $d''(v)=0$; and $E^2_{p,q}=Z^2_{p,q}/B^2_{p,q}$.

The nth group $C_n(\text{tot})$ of the total complex ($n\geqq 0$) is the group $C_{0,n}\oplus C_{1,n-1}\oplus \cdots \oplus C_{n-1,1}\oplus C_{n,0}$. Set $D=d''+d'$, the differential of the total complex $\{C_n(\text{tot})\}$. Denote by Z^n the elements of $C_n(\text{tot})$ of the form $x=(0,\cdots,0,u,v)$ with $Dx=0$. Denote by $\dot{B}^n$ the elements of Z^n of the form $x=Dy$, where $y=(0,\cdots,0,z_1,z_0)\in C_{n-1}(\text{tot})$, and B^n the elements of Z^n of the form $x=Dy$ where $y=(z_n,z_{n-1},\cdots,z_2,z_1,z_0)\in C_{n-1}(\text{tot})$. Then the filtered group $F^{n-1}H^n$ of the total complex associated to the bicomplex is Z^n/B^n. We denote by $\dot{H}^n$ the group $Z^n/\dot{B}^n$. Note that there is clearly an epimorphism from $\dot{H}^n$ onto $F^{n-1}H^n$ for all n, and for $n=2$ or 1 it is the

AMS subject classifications. Primary 1616, 1840; Secondary 1672, 1320, 1810, 1205.

Key words and phrases. Azumaya algebras, bicomplex of abelian groups, Galois cohomology, normal central simple algebras, exact sequence of low degree.