TRUNCATION ERROR BOUNDS FOR π-FRACTIONS

BY W. B. GRAGG[1]

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1. **Preliminaries.** The purpose of this note is to state extensions of the results given in [2] for g-fractions. These extensions will be useful for a unification of the theory of inclusion regions for continued fractions associated with certain Hilbert transforms

$$f(z) = \int_{-\infty}^{+\infty} \frac{d\sigma(t)}{z - t}.$$

For related results see [1], [3], and [4].

For $-\infty < a < b < +\infty$ let $W(a, b)$ denote the class of nonrational real analytic functions $f(z)$ which are holomorphic for $z \in \text{comp}[a, b]$ and which satisfy $\text{Re}[((z-a)(z-b))^{1/2} f(z)] > 0$ in this domain. The principal branch of the square root is assumed.

THEOREM 1. *The following alternative characterizations of the class $W(a, b)$ are valid:*

(a) *$f \in W(a, b)$ if and only if there is a bounded nondecreasing function σ, with infinitely many points of increase, such that*

$$f(z) = \int_a^b \frac{d\sigma(t)}{z - t}, \qquad z \in \text{comp}[a, b];$$

(b) *$f \in W(a, b)$ if and only if f has a (unique) π-fraction expansion*

$$f(z) = \frac{\pi_0 |}{|z - b} + \frac{b - a|}{| 1} + \frac{\pi_1(z - a)|}{| z - b} + \frac{b - a|}{| z - b} + \frac{\pi_2(z - a)|}{| z - b} + \cdots, \qquad z \in \text{comp}[a, b], \tag{1}$$

with $\pi_n > 0$, $n \geqq 0$.

2. **Inclusion regions.** The first inclusion theorem is a consequence of Theorem 1(a).

THEOREM 2. *If $f \in W(a, b)$ and z is nonreal then $f(z)$ is contained in the open convex sector $K_{-1}(z)$ bounded by the rays*

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