

ON THE SUZUKI AND CONWAY GROUPS

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The Suzuki and Conway groups were constructed from the six-dimensional complex representation of a central extension of Z_6 by $PSU_4(3)$ in [2]. In fact, if Q is the rational number field, $\sqrt{-3} = \sqrt{3}i = w - \bar{w}$, $w = (-1 + \sqrt{3}i)/2$, and $w^3 = 1$, then the following unitary matrices M_1, \dots, M_6 generate a central extension H of Z_6 by the Suzuki group of order $2^{18}3^75^{27}(11)(13)$:

$$M_1 = \frac{1}{\sqrt{-3}} \left[\begin{array}{c} \begin{pmatrix} 1 & 1 & 1 \\ 1 & w & \bar{w} \\ 1 & \bar{w} & w \end{pmatrix} \oplus \begin{pmatrix} -1 & -1 & -1 \\ -1 & -\bar{w} & -w \\ -1 & -w & -\bar{w} \end{pmatrix} \\ \oplus \begin{pmatrix} -\bar{w} & -w & -1 \\ -w & -\bar{w} & -1 \\ -1 & -1 & -1 \end{pmatrix} \oplus \begin{pmatrix} 1 & 1 & 1 \\ 1 & w & \bar{w} \\ 1 & \bar{w} & w \end{pmatrix} \right],$$

$$M_2 = \text{diag}(w, w, w, w, w, w, \bar{w}, \bar{w}, \bar{w}, \bar{w}, \bar{w}, \bar{w}).$$

The following denote permutation matrices where our 12 variables are $x_1, \dots, x_6, x_{1'}, \dots, x_{6'}$.

$$M_3 = (1 \ 2 \ 3)(1' \ 2' \ 3')(4' \ 5' \ 6'),$$

$$M_4 = (4 \ 5 \ 6)(1' \ 2' \ 3')(4' \ 6' \ 5'),$$

$$M_5 = (1 \ 2 \ 6)(4' \ 3' \ 2')(1' \ 6' \ 5'),$$

$$M_6 = (1 \ 2 \ 1')(5 \ 4' \ 4)(6 \ 5' \ 6').$$

A lattice \mathcal{L} fixed by H can be defined in terms of the following partitions $\{1, \dots, 6'\} = S_i \cup C(S_i)$, $i=1, 11$. These partitions are permuted by the above permutation matrices:

$$S_1 = \{1, 2, 3, 4, 5, 6\},$$

$$S_2 = \{1, 2, 3, 1', 2', 3'\},$$

$$S_3 = \{1, 2, 4, 2', 4', 5'\},$$

$$S_4 = \{1, 2, 5, 3', 4', 6'\},$$

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