K-THEORETIC INTERPRETATION OF TAME SYMBOLS ON $k(t)$

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Communicated by Hyman Bass, April 16, 1970

In [3] we introduced a canonical resolution for computing the K-theory of [4] and we found a map $\psi: K_2(A) \to \kappa_2^{GL}(A)$ where $K_2(A)$ is the group defined by Milnor [5] and $\kappa_2^{GL}(A)$ is the group of [3]. The map ψ was proved surjective if A is a regular ring. In this announcement we indicated how to compute $\kappa_2^{GL}(k(t))$ for the field $k(t)$ of rational functions in one variable t. As a biproduct of this work we have proved

THEOREM 1. *Write* $K_2(A[t, t^{-1}]) = K_2(A) \oplus X$. *Then if A is regular, X has a homomorphic image* $K_1(A)$.

I should like to thank H. Bass for suggesting that Theorem 1, which was buried in my original announcement, be set off as a main result. Bass has informed me that J. Wagoner also has results on the group X.

1. **Generalities.** If R is any ring (without unit) recall the path ring $\Omega R = x(1-x)R[x]$. Clearly $\Omega(R[T]) = (\Omega R)[T]$ if T is a free abelian group or monoid. Also $\kappa_2^{GL}(R) \cong \kappa_1^{GL}(\Omega R)$ [3].

PROPOSITION 1[1]. $\kappa_1^{GL}(R[t]) = \kappa_1^{GL}(R)$ *and* $\kappa_1^{GL}(R[t,t^{-1}]) = \kappa_1^{GL}(R) \oplus \overline{K}_0(R^+)$.

This is an easy consequence of results of [1] and [3].

PROPOSITION 2. *If A is regular, then the composition ω is a surjective homomorphism.*

$$\begin{array}{ccccc} K_2(A[t,t^{-1}]) & \longrightarrow & \kappa_2^{GL}(A[t,t^{-1}]) & \xrightarrow{\cong} & \kappa_1^{GL}(\Omega A[t,t^{-1}]) \\ & \searrow^{\omega} & & & \downarrow \\ & & & & \overline{K}_0((\Omega A)^+) \end{array}$$

Theorem 1 follows from this proposition using results of [1] and [5].

AMS 1970 *subject classifications*. Primary 13D15, 16A54, 16A62.

Key words and phrases. K-theory, tame symbol, discrete valuation ring.

[1] ADDED IN PROOF. For the second conclusion of Proposition 1 we require that R be of the form $\Omega^n S$ where S is regular.