SOME RESULTS ON TOPOLOGICAL NEIGHBOURHOODS

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Full proofs of results announced here are to be found in [**10**]. We consider the problem of classifying germs of neighbourhoods of locally flat topological submanifolds. Let M^n be a fixed topological manifold. An *r-neighbourhood* of M is a pair (i, N) where N is a topological manifold of dimension $n+r$ and $i: M \to N$ a locally flat embedding satisfying $i^{-1}(\partial N) = \partial M$. Denote the set of germs of r-neighbourhoods (i.e. equivalence classes under homeomorphism defined near $i(M)$ and fixing M) by $\mathfrak{N}_r(M)$.

MAIN THEOREM. *There is a countable* CW *complex* B $\mathbf{Top}_r$ *and a function* $c: \mathfrak{N}_r(M) \to [M, B\ \mathbf{Top}_r]$, *which is a bijection if* $\partial M = \emptyset$ *and* $(n, r) \neq (1, 3)$, *or if* $\partial M \neq \emptyset$ *and* $(n, r) \neq (1, 3)$ *or* $(2, 3)$.

The omitted cases in the theorem are due to the unsolved 4-dimensional annulus conjecture; an extension to any one of these cases is strictly equivalent to the conjecture.

OUTLINE OF PROOF. We rework Haefliger's theory of microbundle pairs [2] in the topological category. The crucial tool is immersion theory. An *r-microbundle pair* is a pair $\epsilon^k \subset \zeta^{k+r}$ of microbundles, where ϵ^k denotes the trivial bundle of rank k and the inclusion is locally trivial. Two such are *equivalent* (stably) if they are isomorphic after possibly adding further trivial bundles to both elements and the isomorphism is the identity on the trivial sub-bundle. The set of equivalence classes forms a good 'theory'; the group is $\mathbf{Top}_r = \lim_{k\to\infty} \{\mathrm{Top}_{r+k,k}\}$, where $\mathrm{Top}_{r+k,k} = \{$germs of automorphisms of R^{r+k} fixing $\{0\} \times R^k\}$; the classifying space is $B\ \mathbf{Top}_r$. We associate to the neighbourhood (i, N) the pair $\tau_M \oplus \nu \subset i^*(\tau_N) \oplus \nu$, where ν is a stable inverse to τ_M and this defines the function c. Using immersion theory we show that a germ of neighbourhood is equivalent to a pair $\tau_M \subset \xi^{n+r}$ and then the theorem follows by diagram chasing from:

FIRST STABILITY THEOREM. $\pi_i(\mathrm{Top}_{r+k,k}) \to \pi_i(\mathbf{Top}_r)$ *is an isomorphism provided* $i \leq k$ *and either* $k+r \geq 5$ *or* $r \leq 2$.

This follows from the PL result (Haefliger [2], see also [**11**, III 5.4]) and the analogous result for $\mathrm{Top}_{r+k,k}/\mathrm{PL}_{r+k,k}$ which, using

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