COMPLETELY REGULAR MAPPINGS AND DIMENSION[1]

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1. **Introduction.** In an earlier paper [12] the author proved the following theorem: There exists a monotone open map of the universal curve onto any continuous curve such that each point-inverse set is also a universal curve. Since these mappings are open and have homeomorphic point-inverse sets, it is natural to ask whether or not these mappings are completely regular. Theorem 1 of this paper shows that they will be completely regular only if the range is a point. Theorem 1, Theorem 3, and the corollary to Theorem 3 all give conditions on completely regular mappings so that they will not raise dimension. Theorem 4 actually classifies completely regular mappings of a certain type.

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2. **The main theorem.**

THEOREM 1. *If f is a completely regular mapping of an n-dimensional compactum X onto a compactum Y and $\check{H}^n(f^{-1}(y)) \neq 0$ for all $y \in Y$, then Y is 0-dimensional.*

LEMMA 1. *Let X be an n-dimensional compactum. Let J be a finite polyhedron contained in E^{2n+1} of dimension less than $n+1$. If f is a mapping of X into E^{2n+1} and $\eta > 0$, then there exists a homeomorphism $h: X \to E^{2n+1}$ such that $d(f, h) < \eta$ and $h(X) \cap J = \emptyset$.*

PROOF OF LEMMA 1. Approximate f by a mapping g whose range is contained in an n-polyhedron which (by general positioning) misses J. Since the set of homeomorphisms is dense in the function space $(E^{2n+1})^X$, we can find a homeomorphism h which approximates g and such that $h(X) \cap J = \emptyset$.

The homology theory in this paper will be singular homology with integer coefficients. If J is a singular n-cycle, then $|J|$ will denote its

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