DUALITY OF MULTIPLICATIVE FUNCTIONALS

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1. **Introduction.** Suppose X and $\hat{X}$ is a pair of standard processes in duality relative to a Radon measure ξ. We refer the reader to [**1**] for all terminology and notation not explicitly defined here. In particular (U^α) and $(\hat{U}^\alpha)$ denote the resolvents of X and $\hat{X}$ respectively and the α-potential kernel $u^\alpha(x, y)$ satisfies

$$U^\alpha(x, dy) = u^\alpha(x, y)dy, \qquad \hat{U}^\alpha(x, dy) = u^\alpha(y, x)dy.$$

Here $dy = \xi(dy)$. We make no regularity assumptions on the resolvents of X and $\hat{X}$. One of the most important properties of such dual processes is (VI-1.16) (all such references are to [**1**]) which states that if A is a Borel set then for all $\alpha \geqq 0$ and x, y

$$P_A^\alpha u^\alpha(x, y) = u^\alpha \hat{P}_A^\alpha(x, y). \tag{1.1}$$

This result which is due to Hunt says that the process X killed at the time it first hits A and the process $\hat{X}$ killed when it first hits A are in duality. In particular if we define

$$Q_t f(x) = E^x\{f(X_t); t < T_A\} \quad \text{and} \quad \hat{Q}_t f(x) = \hat{E}^x\{f(X_t); t < T_A\}$$

(for typographical reasons we will omit the hat "^" in those places where it is obviously required—see the remark on p. 262 of [**1**]), then it is a standard observation that (1.1) is equivalent to

$$(Q_t f, g) = (f, \hat{Q}_t g) \tag{1.2}$$

for all $t \geqq 0$ and for all continuous functions with compact support, f and g. Here $(\phi, \psi) = \int \phi(x)\psi(x)dx$.

The purpose of this paper is to announce an extension of (1.2) and (1.1) to a more general class of multiplicative functionals than those of the form $M_t = I_{[0,T_A)}(t)$. Our basic result is that if M is an exact MF (multiplicative functional) of X then there exists a unique exact MF, $\hat{M}$, of $\hat{X}$ such that (1.2) holds where $\{Q_t\}$ and $\{\hat{Q}_t\}$ are the semigroups generated by M and $\hat{M}$ respectively and that an appropriate

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