CROSS SECTIONALLY CONNECTED 2-SPHERES ARE TAME

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Communicated by Steve Armentrout, March 5, 1970

W. T. Eaton [4] and Norman Hosay [5] have independently shown that a 2-sphere S in E^3 is tame if each horizontal cross section of S is either a simple closed curve or a point. The purpose of this note is to indicate how to extend Hosay's argument to show that S is tame if each horizontal cross section is connected. This answers a question raised by Bing [2].

The author would like to thank L. D. Loveland for helpful suggestions.

The notation used here is as in [5]. Let $E_t = \{(x, y, z) \in E^3 | z = t\}$.

THEOREM. *Let S be a 2-sphere in E^3 such that $S \cap E_t$ is connected (or void) for each t in E^1. Then S is tame.*

Let $J_t = S \cap E_t$. We suppose $\{t | J_t \neq \varnothing\} = [0, 1]$. The first four parts of Hosay's proof are concerned with showing that S is locally tame modulo $J_0 \cup J_1$ by showing that the complementary domains of S are locally simply connected at each point p of $S - (J_0 \cup J_1)$. For a round open ball U containing p he picks a certain map h taking a disk D into $U \cap \text{Cl}(\text{Int } S)$ and wishes to construct a map $g: D \to U - S$ which agrees with h on Bd D.

We first observe that since a separable metric space can contain only countably many mutually disjoint separators which are not irreducible, the set J_t, $0 < t < 1$, is an irreducible separator of S (and hence of E_t) except for at most countably many values of t. Using Cannon's result [3] we know that each set J_t, $0 < t < 1$, is a taming set. We next observe that if $\{J_i\}$ is a countable collection of taming sets on S the techniques of [1] can be used to construct an ϵ-map of Cl(Int S) into Cl(Int S) $- \cup J_i$. (Proofs of these observations appear in [6].) Thus we may suppose that $h(D) \cap J_t = \varnothing$ unless J_t is an irreducible separator of E_t. This is the key to extending Hosay's argument.

In part (A) of [5] Hosay uses the fact that if $h(A_i^t)$ is a certain continuum in $h(D) \cap E_t$ then any two points of $h(A_i^t) \cap \text{Int } S$ can be

AMS 1969 *subject classifications*. Primary 5705; Secondary 5478.

Key words and phrases. Tame 2-spheres, tame surfaces, surfaces in E^3.

[1] The results presented in this paper are a part of the author's dissertation at the University of Wisconsin, written under the direction of R. H. Bing.