ASYMPTOTICS FOR $\Box u = m^2u + G(x, t, u, u_t, u_x)$. I. GLOBAL EXISTENCE AND DECAY[1]

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The existence of global solutions to equations of the form

$$\Box u = m^2u + G(x, t, u, u_t, u_x), m > 0,$$

can be proved for a wide class of perturbations G. Estimates on the decay of these solutions (i.e. of

$$\|u(t)\|_r = \|u(\cdot, t)\|_r \quad \text{and} \quad \|\dot{u}(t)\|_r = \|u_t(\cdot, t)\|_r$$

as $|t| \to \infty$) which are suitable for the scattering theory of these equations have also been obtained. The results to be summarized here generalize the decay results of Segal [1] for $G(u)$ not only in that more general types of perturbations may be treated but also in the fact that no a priori global existence is required.

1. Abstract decay result. Let A^2 denote the selfadjoint realization of $m^2I - \Delta$ on $L^2(E^n)$. The real solution spaces, $H(A, a)$, which are relevant in this work are, for each $a \in R$, the completions of $D(A^a) \oplus D(A^{a-1})$ with respect to the inner product

$$\left(\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}\right)_{A,a} := (A^a u_1, A^a v_1) + (A^{a-1}u_2, A^{a-1}v_2).$$

The norm of $\binom{u(t)}{\dot{u}(t)} \in H(A, a)$ will be denoted by $|u(t)|_a$ as opposed to the usual L^p-norm $\|u(t)\|_p$, and $G(\cdot, t, u(t), \dot{u}(t), u_x(t))$ will be replaced by $G(t, u(t))$.

A list of assumptions will now be presented leading up to the final result which will be given as a summarizing theorem. To begin, pick r and a in such a way that

$$\text{(I)} \qquad \|u(t)\|_r, \|\dot{u}(t)\|_r \leqq \text{Const} \, |u(t)|_a,$$

so that the continuity of $\|u(t)\|_r$ and $\|\dot{u}(t)\|_r$ follows from that of

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