A CLASS OF PERFECT DETERMINANTAL IDEALS

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In recent years several authors [1], [2], [4], [13], [17], [18] have studied the special homological properties of ideals generated by the subdeterminants of a matrix or "determinantal" ideals. The question of whether the ideal of $m+1$ by $m+1$ minors of an r by s matrix is perfect if the grade is as large as possible, $(r-m)(s-m)$, has remained open, although the special cases $m=0$, 1, and $r-1$ $(r\leqq s)$ are known. The general result is Corollary 4 of Theorem 1. For purposes of the induction argument used to prove the theorem it is necessary to consider a larger class of ideals somewhat complicated to describe.

THEOREM 1. *Let R be a commutative Noetherian ring with identity. Let $M=(c_{ij})$ be an r by s matrix with entries in R. Let $H=(s_0, \cdots, s_m)$ be a strictly increasing sequence of nonnegative integers such that $s_0=0$, $s_m=s$, and $m<r$. Let n be an integer, $0\leqq n\leqq s$. Let $I=I_{H,n}=I_{H,n}(M)$ be the ideal of R generated by the $t+1$ by $t+1$ minors of the first s_t columns of M, $1\leqq t\leqq m$, and $c_{11}, \cdots, c_{1n}$. Let h be the least integer such that $s_h\geqq n$. Suppose that the grade of (i.e. the length of the longest R-sequence contained in) I is as large as possible, namely*

$$g = g_{H,n} = rs - (r+s)m + h + \frac{1}{2}m(m+1) + s_1 + \cdots + s_{m-1}.$$

Then $I_{H,n}$ is perfect in the sense of Rees, that is, the homological (or projective) dimension of R/I over R is also equal to g.

COROLLARY 1. *If $I_{H,n}$ has grade $g_{H,n}$ then it is grade unmixed, i.e. the associated primes of $I_{H,n}$ all have grade $g_{H,n}$.*

COROLLARY 2. *If R is Cohen-Macaulay (locally), and $I_{H,n}$ has grade $g_{H,n}$, then $I_{H,n}$ is rank unmixed, i.e. the associated primes all have rank ($\equiv$ altitude) $g_{H,n}$; moreover, R/I is Cohen-Macaulay.*

COROLLARY 3. *The rank of any minimal prime of $I_{H,n}$ is at most $g_{H,n}$ (with no conditions on the grade of I).*

COROLLARY 4. *When $H=(0, 1, 2, \cdots, m-1, s)$ and $n=0$, $I_{H,n}$ is*

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