ON THE GALOIS THEORY OF PURELY INSEPARABLE FIELD EXTENSIONS

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The main purpose of this announcement is to show that those purely inseparable field extensions which behave in a certain sense like normal extensions in fact are of a fundamentally abelian character. Detailed proofs of most results are contained in the second author's thesis [6].

1. **Exponent 1.** Throughout K will be a finite purely inseparable extension of a field k of characteristic p and Der K/k will denote the K-space of derivations of K over k. We consider first the case where K/k has exponent one. In that case we have

THEOREM 1. *Suppose that $\phi_1, \cdots, \phi_n$ are commuting derivations of K over k which are linearly independent over k. Then*

1. *They are independent over K.*
2. $[K:k] \geqq n$.
3. *Equality holds iff the k-space V_0 spanned by $\phi_1, \cdots, \phi_n$ is closed under the formation of pth powers, in which case $V_0 \otimes_k K = \mathrm{Der}\ K/k$.*

Let us call a K-subspace V of Der K/k **restricted** if $\phi \in V$ implies $\phi^p \in V$. From Theorem 1 it is then easy to deduce that:

(i) every restricted subspace of Der K/k is spanned by commuting derivations, and

(ii) every restricted K-subspace V of Der K/k is of the form Der K/L for some unique intermediate field $k \leqq L \leqq K$.

The latter assertion, an exact analog of the fundamental theorem of the Galois theory for purely inseparable extensions of exponent one, was first proved by Jacobson [2] under the additional hypothesis that V is a Lie subalgebra of Der K/k. The stronger form is due to Gerstenhaber [4]. One sees a posteriori that a restricted subspace is necessarily a Lie subalgebra.

The three parts of Theorem 1 are precisely analogous to Theorems 12, 13, and 14 of [1], by means of which Artin demonstrates the usual "fundamental theorem" of the Galois theory.

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