

## NEW SPHERE PACKINGS IN DIMENSIONS 9-15

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New sphere packings in Euclidean spaces  $E^9$  to  $E^{15}$  are constructed using nonlinear error-correcting codes and Steiner systems. The new packings and the previously best packings are collected in Table 1, which thus supersedes part of the table given by Leech [1].

**1. Sphere packings from distance 4 codes.** Let an  $(n, M, d)$  error-correcting code be a set of  $M$  binary vectors of length  $n$  such that any two vectors differ in at least  $d$  places. Given such a code with  $d = 4$  a packing of unit spheres in  $E^n$  may be obtained by the following

*Construction.*  $\mathbf{x} = (x_1, \dots, x_n)$  is a center of the packing if and only if  $\mathbf{x}$  is componentwise congruent modulo 2 to a vector in the code.

Let the center density of a packing denote the fraction of space covered by the spheres divided by the content of a unit sphere. It is easily seen that this packing has center density  $M2^{-n}$  and that the number of spheres touching the sphere with center congruent to a codeword  $\mathbf{c}$  is equal to  $2n + 16A(\mathbf{c})$ , where  $A(\mathbf{c})$  is the number of codewords differing from  $\mathbf{c}$  in exactly four places. The codes used here are nonlinear codes and so the packings obtained are nonlattice packings.

Nonlinear single error-correcting  $(8, 20, 3)$ ,  $(9, 38, 3)$ ,  $(10, 72, 3)$  and  $(11, 144, 3)$  codes have been given by Golay [2] and Julin [3]. By annexing a 0 at the end of codewords of even weight and a 1 at the end of codewords of odd weight they are made into codes with  $d = 4$ . The packings  $P9a$ ,  $P10a$ ,  $P11a$ ,  $P12a$  of Table 1 are then obtained by applying the above construction to these codes. The codes are not unique and several inequivalent versions of the packings are possible.

It is worth remarking that these codes have more codewords (i.e., are more densely packed!) than any group code of the same length and minimum distance. They have a simple structure, and have recently been generalized to give single error-correcting codes of all lengths  $2^m \leq n < 3 \cdot 2^{m-1}$ ,  $m \geq 3$ , having more codewords than any comparable group code [4], [5].

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