NONLINEAR ELLIPTIC BOUNDARY VALUE PROBLEMS AND THE GENERALIZED TOPOLOGICAL DEGREE

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Introduction. It is our purpose in the present note to present a general existence theorem for noncoercive elliptic boundary value problems for operators of the form:

$$A(u) = \sum_{|\alpha| \leq m} (-1)^{|\alpha|} D^\alpha A_\alpha(x, u, \cdots, D^m u), \tag{1}$$

on closed subspaces V of the Sobolev space $W^{m,p}(G)$, G an open subset of R^n, $n \geqq 1$. This existence theorem is based upon an extension of the theory of the generalized topological degree for A-proper mappings of Banach spaces introduced in Browder-Petryshyn [8], [9], and, in particular, on an extension of the Borsuk-Ulam theorem to pseudomonotone mappings T from a reflexive separable Banach space V to its conjugate space V^*.

To make a precise statement of our general existence theorem possible, we introduce the following notation: For a given $m \geqq 1$, we let ξ be the m-jet of a function u from R^n to R^s for some given $s \geqq 1$, i.e. $\xi = \{\xi_\alpha : |\alpha| \leqq m\}$, and set

$$\zeta = \{\zeta_\alpha : |\alpha| = m\}, \qquad \eta = \{\eta_\beta : |\beta| \leqq m-1\},$$

where each ξ_α, ζ_α, and η_β is an element of R^s. The set of all ξ of the above form is an Euclidean space R^{r_m}, and correspondingly, $\zeta \in R^{r_m'}$, $\eta \in R^{r_{m-1}}$.

For each α, A_α is assumed to be a function from $G \times R^{r_m}$ to R^s satisfying the following conditions:

Assumptions on $A(u)$: (1) $A_\alpha(x, \xi)$ *is measurable in* x *for fixed* ξ *and continuous in* ξ *for fixed* x. *For a given* p *with* $1 < p < \infty$, *there exists a constant* c *such that*

$$|A_\alpha(x, \xi)| \leqq c\left((1 + \sum_{|\beta| \leq m} |\xi_\beta|^{p_{\alpha\beta}}\right)$$

with $p_{\alpha\beta} \leqq (p-1)$ *for* $|\alpha| = |\beta| = m$, *and*

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