

## RESEARCH ANNOUNCEMENTS

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### GROUP INVARIANCE IN NONLINEAR FUNCTIONAL ANALYSIS

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**Introduction.** If  $X$  is an infinite dimensional Banach space (or more generally, an infinite dimensional manifold),  $T$  and  $S$  two mappings of  $X$  into another space  $Y$ , the typical problems of nonlinear functional analysis ask about the set of points  $u$  in  $X$  for which  $T(u) = S(u)$ , or for which  $T(u) = \lambda S(u)$  for some real  $\lambda$ , or for which  $T(u) = y_0$  for a given  $y_0$  in  $Y$ . Aside from basic structural hypotheses on the classes of mappings  $T$  and  $S$  considered (i.e. hypotheses that one operator or the other is compact, monotone, accretive, nonexpansive, proper, Fredholm, or whatever), in order to obtain nontrivial existence results for the desired solutions  $u$ , one must impose additional hypotheses in the large usually in the form of boundary conditions or asymptotic conditions (coerciveness, boundedness of inverse mappings, etc.). There is an alternative type of additional hypothesis, however, under which one obtains nontrivial results with the boundary or asymptotic conditions weakened or eliminated, namely the hypothesis that the nonlinear problem is invariant under a group  $G$  of transformations acting on the spaces  $X$  and  $Y$  with  $G$  having elements of finite order.

In another paper (Browder [5]), we have obtained results on the application of the Lusternik-Schnirelman theory to obtain infinitely many distinct solutions of the nonlinear eigenvalue problem  $g'(u) = \lambda h'(u)$ , where  $g'$  and  $h'$  are the Fréchet derivatives of real-valued functions  $g$  and  $h$  on an infinite dimensional Banach space  $B$  and the usual hypothesis that  $g$  and  $h$  are even functions is replaced by the

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