ACYCLICITY IN THREE-MANIFOLDS

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ABSTRACT. An acyclic compactum in an orientable, open 3-manifold has arbitrarily close, polyhedral neighborhoods whose components are compact 3-manifolds with a special structure. Frequently, these 3-manifolds have free fundamental groups. These observations and some results from combinatorial group theory are exploited to deduce facts about the homomorphism of fundamental groups induced by an acyclic mapping. The techniques are applied to relate local homotopy properties of quotient spaces of acyclic upper semicontinuous decompositions, to "UV" (or "shape") properties of the elements in the decomposition. It is shown that a "0-dimensional" monotone decomposition of Euclidean k-space is acyclic if the quotient space is an open k-manifold. (For $k=3$, such a decomposition is shown to be cellular.) Some conditions are given under which acyclic decompositions are cellular.

1. **Introduction.** Let G be an upper semicontinuous decomposition of Euclidean 3-space E^3, into compact, connected sets such that for some prime p, each $g \in G$ is strongly 1-acyclic over Z_p (the integers modulo p). Our purpose is to show that some useful information of a homotopy-theoretic nature about the decomposition space E^3/G and the projection mapping P_G, can be deduced from an examination of the nondegenerate elements of G (whose union is denoted H_G). For example, we prove that a necessary condition that E^3/G should be locally simply connected at $P_G(g)$, is that g should have property 1-UV. (These and other terms are defined later.)

In particular, if X is a continuum in E^3 which is strongly 1-acyclic over Z_p, then E^3 "modulo" X is locally simply connected if and only if X has property 1-UV. This agrees with R. H. Bing's announcement [7] that E^3 modulo a solenoid is not simply connected and hence not locally simply connected. In light of the 1-dimensional continuum of

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